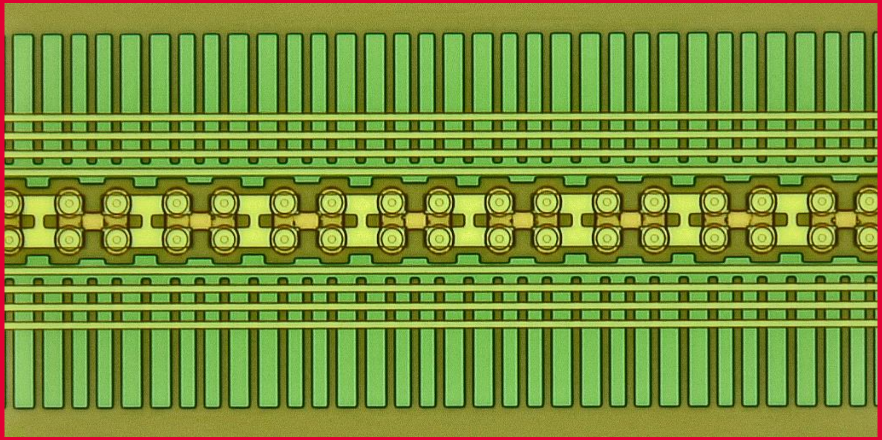
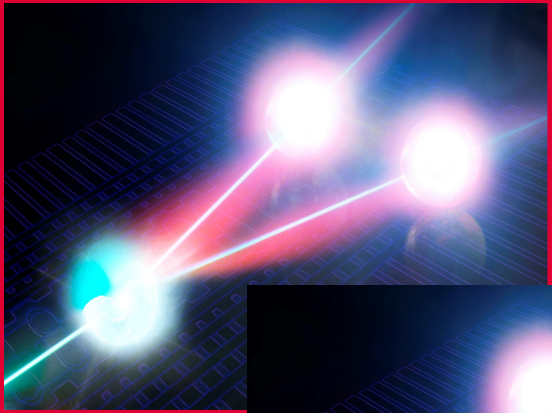
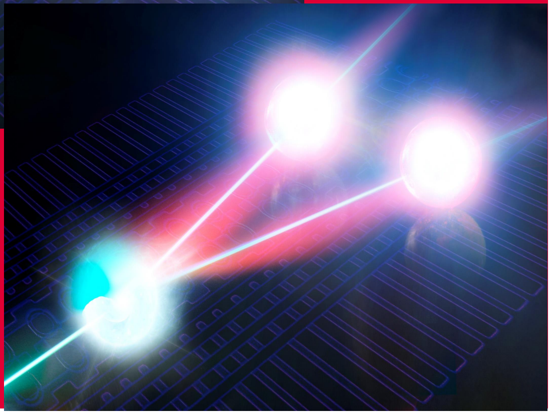
Aalto-yliopisto
Perustieteiden korkeakoulu

# Quantum vacuum, noise, and entanglement

*Pertti Hakonen*
Gdansk, July 10, 2018







# OUTLINE

## Introduction
- Concept of vacuum
- Mode correlations in quantum optics
- Entanglement

## Dynamical Casimir effect
- Photon generation with a Josephson metamaterial
- Correlations: two mode squeezing

## Vacuum fluctuations under double parametric pumping
- New kind of correlations
- Which color information

## Relativity and quantum noise
- Past – future correlations from 4-dim spacetime

## Summary of open problems

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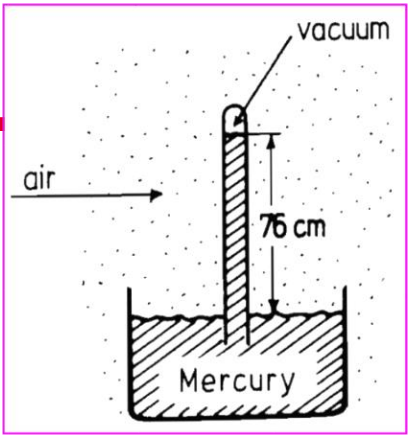
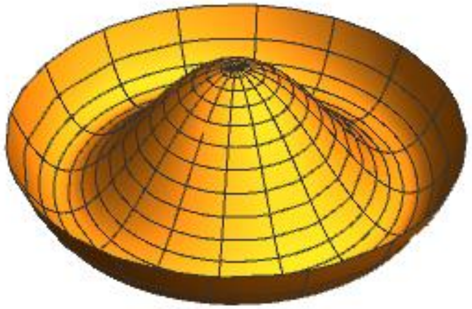
# Introductory remarks



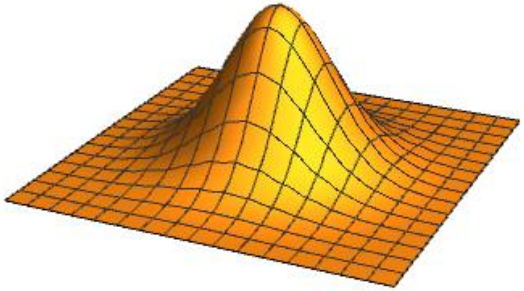

- **Toricellian vacuum**
(Evangelista Toricelli, 1643)

- **First vacuum pump, Magdeburg hemispheres**
(Otto von Guericke, 1654)

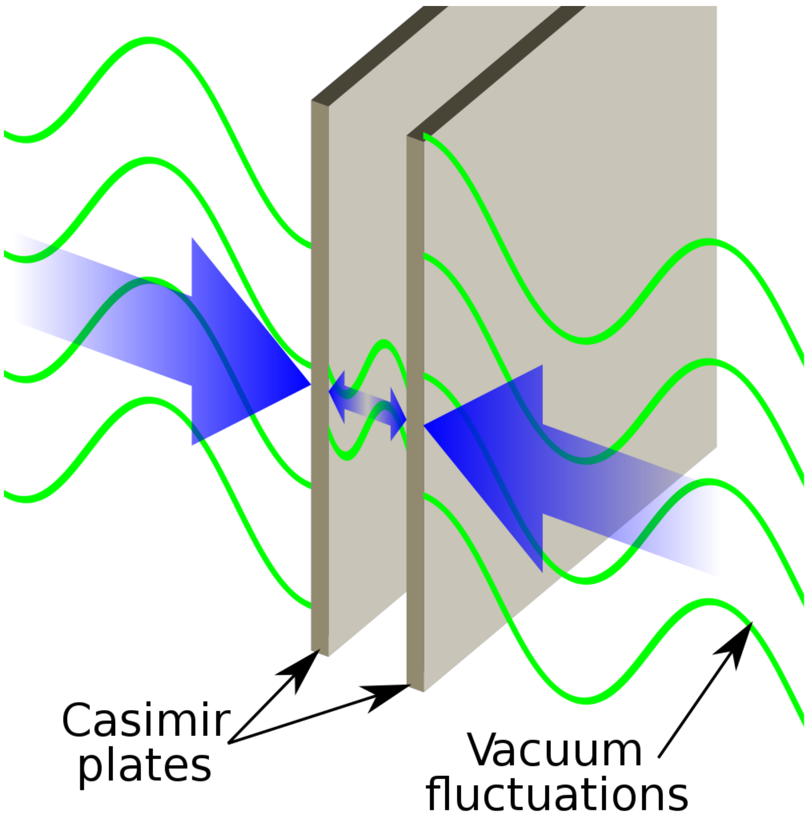
**Modern view of vacuum**
= quantum-mechanical ground state of a field
- Higgs vacuum
- BEC vacuum
- virtual particles, fluctuations



**Effects related to vacuum:**
- spontaneous emission
- Lamb shift
- static Casimir effect



# Vacuum fluctuations: Casimir force




$$\frac{F_C}{A} = -\frac{d}{da}\frac{\langle E \rangle}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$$

“Two ships should not be moored too close together because they are attracted one towards the other by a certain force of attraction.”

The Album of the Mariner
P. C. Caussée,1836

Nature, doi:10.1038/news060501-7

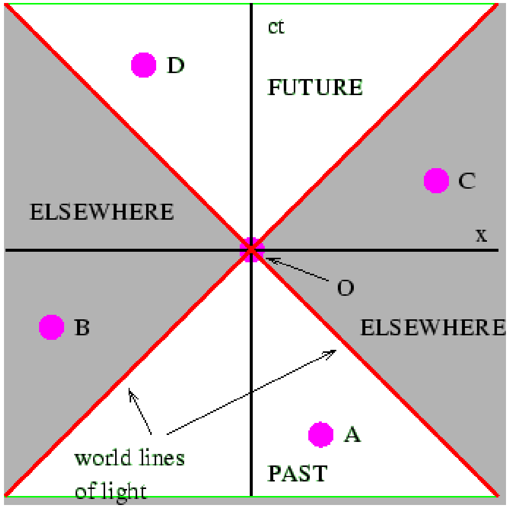
Measured by S. K. Lamoreaux in 1997

# Exciting the vacuum

How to get something out of vacuum:
- use strong electric fields [Schwinger effect]
- change fast a boundary condition or the speed of light [dynamical Casimir effect]
- use a strong gravitational field [Hawking effect]
- accelerate the system [Unruh effect]

- Entanglement of virtual particles
- Entanglement transfer to qubits

- Past-future correlations



# “Mode” observables: Quadratures

$$H_{\mathrm{k}} = \hbar\omega_{\mathrm{k}}(a_{\mathrm{k}}^{\dagger}a_{\mathrm{k}} + \frac{1}{2}))$$

**Quadrature operators (like x and p)*:***

$$X_1 = \frac{1}{\sqrt{2}}\left(a^{\dagger} + a\right)$$

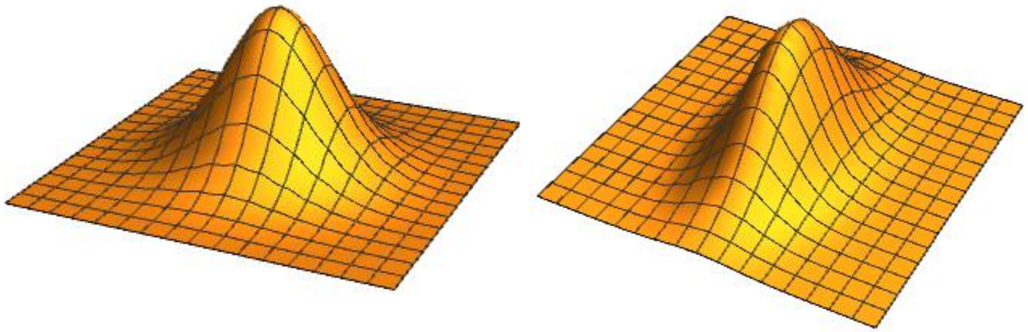
$$X_2 = \frac{i}{\sqrt{2}}\left(a^{\dagger} - a\right)$$

$$X_{\theta} = \frac{1}{\sqrt{2}}\left(ae^{-i\theta} + a^{\dagger}e^{i\theta}\right)$$

**Since** $\left[X_1, X_2\right] = i$**, there must be an uncertainty relation**

$$\Delta X_1 \Delta X_2 \geq \frac{1}{2}$$

**Correlation of quadratures can be manipulated**

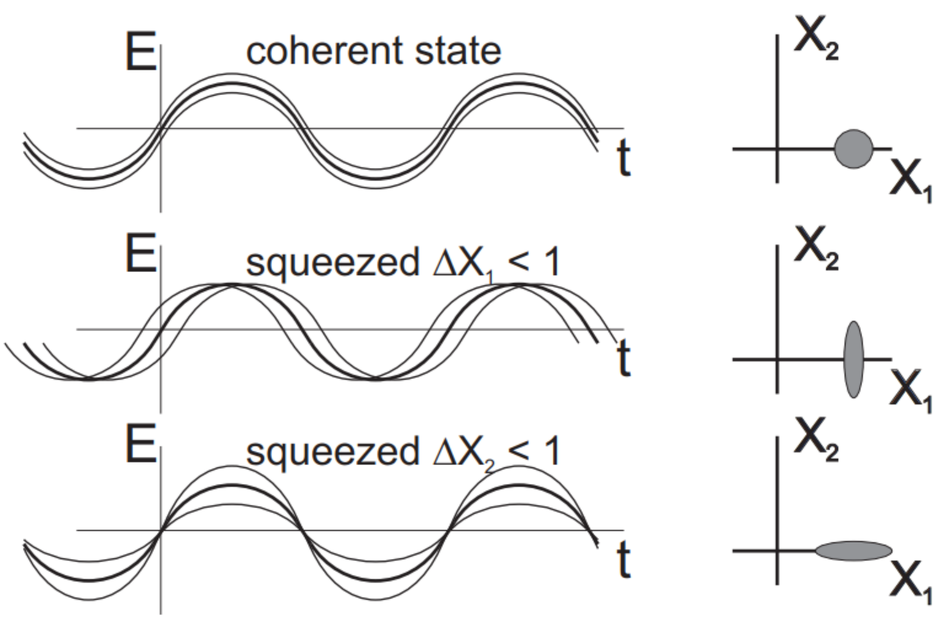


# Single mode squeezing

## Squeezing operator

$$S = \exp\left(\frac{1}{2}\xi a^{\dagger 2} - \frac{1}{2}\xi^* a^2\right) \qquad \xi = re^{i\theta} \qquad |\xi\rangle = S|0\rangle$$

$$\left.\begin{aligned}\langle \Delta X_1{}^2\rangle &= \frac{1}{2}e^{2r} \\ \langle \Delta X_2{}^2\rangle &= \frac{1}{2}e^{-2r}\end{aligned}\right\} \Delta X_1 \Delta X_2 = \frac{1}{2}$$




Basic correlator:

$$\langle aa\rangle = \cosh r \sinh r e^{i\theta}$$

# Two-mode squeezing

## Two mode squeezing operator

$$S_2 = \exp\left(\xi^* ab - \xi a^\dagger b^\dagger\right) \qquad \xi = re^{i\theta}$$

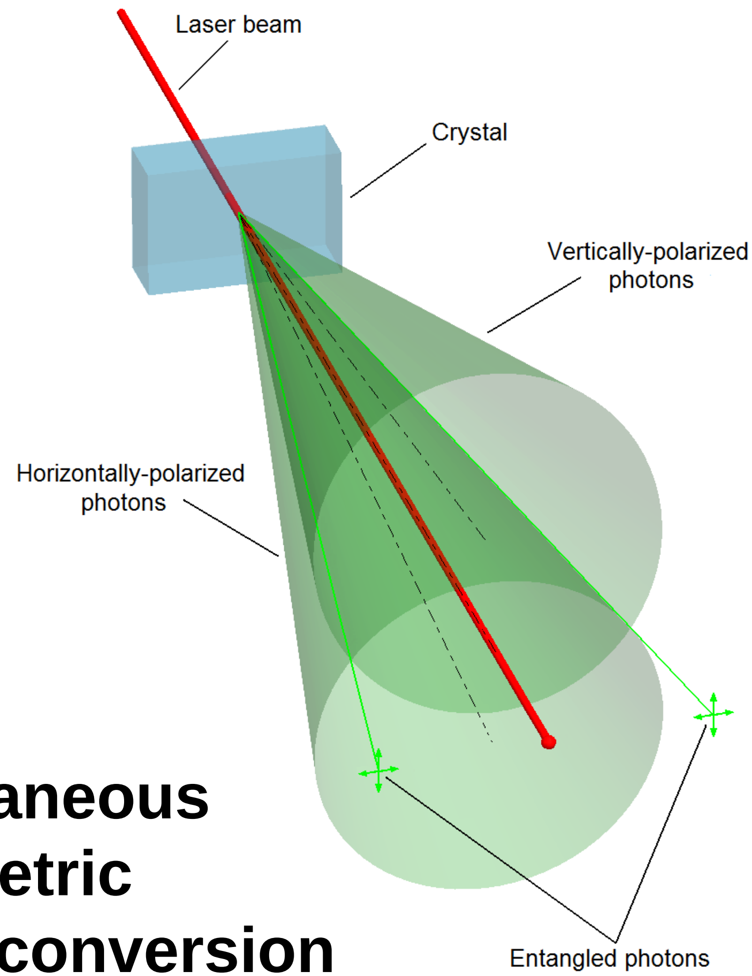
$$\langle ab \rangle = \cosh r \sinh r e^{i\theta} \qquad \langle ab^\dagger \rangle = 0$$

## Maps to single mode case by defining operator

$$d = \frac{1}{\sqrt{2}}(a+b) \qquad \left[d, d^\dagger\right] = 1$$

$$X_\theta^d = \frac{1}{\sqrt{2}}\left(de^{-i\theta} + d^\dagger e^{i\theta}\right) \quad \left\langle \Delta X_1^{d\,2} \right\rangle = \frac{1}{2} e^{2r} \quad \left\langle \Delta X_2^{d\,2} \right\rangle = \frac{1}{2} e^{-2r}$$

# Entanglement




**Spontaneous parametric down-conversion**

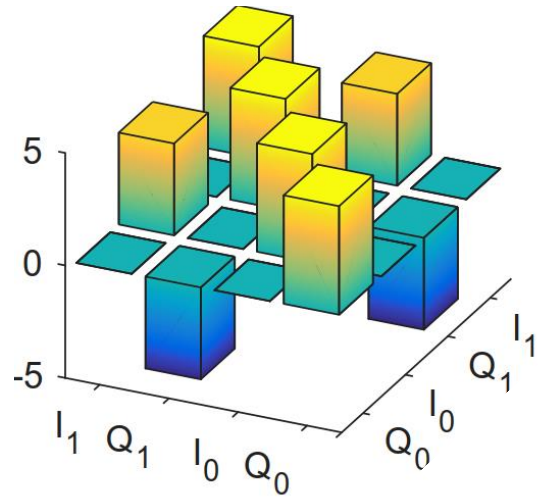
**Polarization in optics**

- vertically/horizontally

$$\left[|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2\right]/\sqrt{2}$$

**Quadratures at microwaves**

- in phase and out of phase

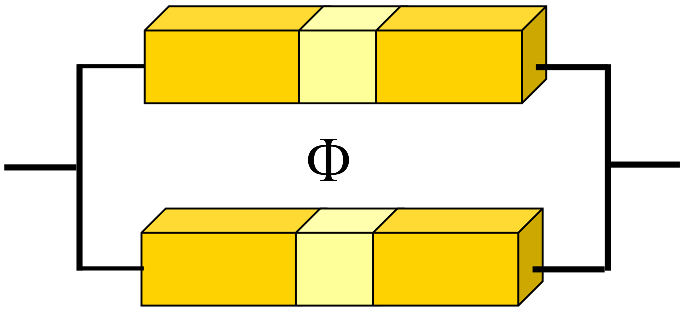



$$\sigma(x_i,x_j)=\langle x_i x_j\rangle$$

**Quantum entanglement:**

$$\sigma(x_+,x_+)-\sigma(x_+,x_-)<1/4$$

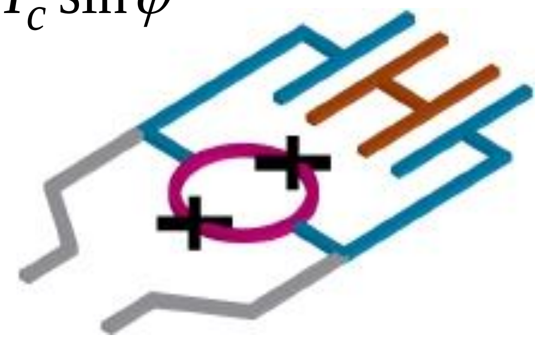
# SQUID: A NONLINEAR *L*

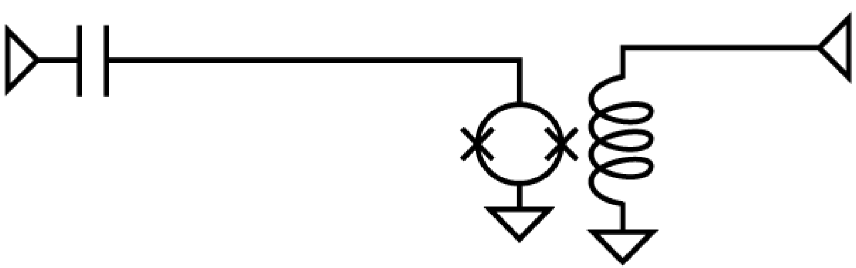



SQUID loop with

$$\varphi = \pi \frac{\Phi}{\Phi_0}$$

$$I = I_c \sin\varphi$$

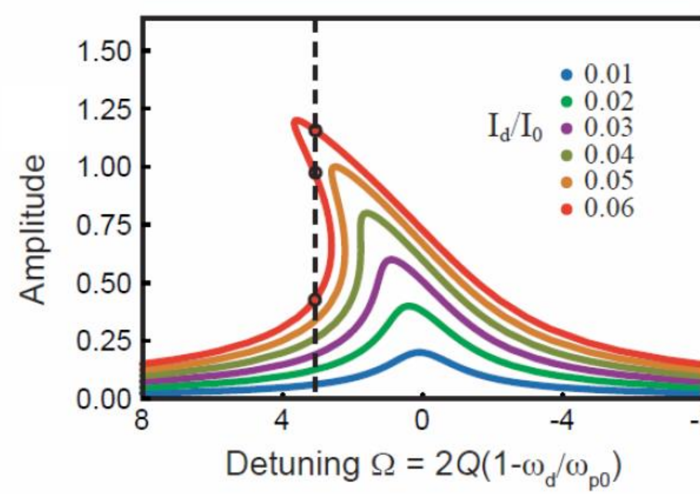




Josephson inductance

$$\frac{1}{L} = \left(\frac{2e}{\hbar}\right)^2 \frac{\partial^2 E}{\partial \varphi^2}$$

$$L_J = \frac{\hbar}{2eI_C} \frac{1}{\cos\varphi}$$



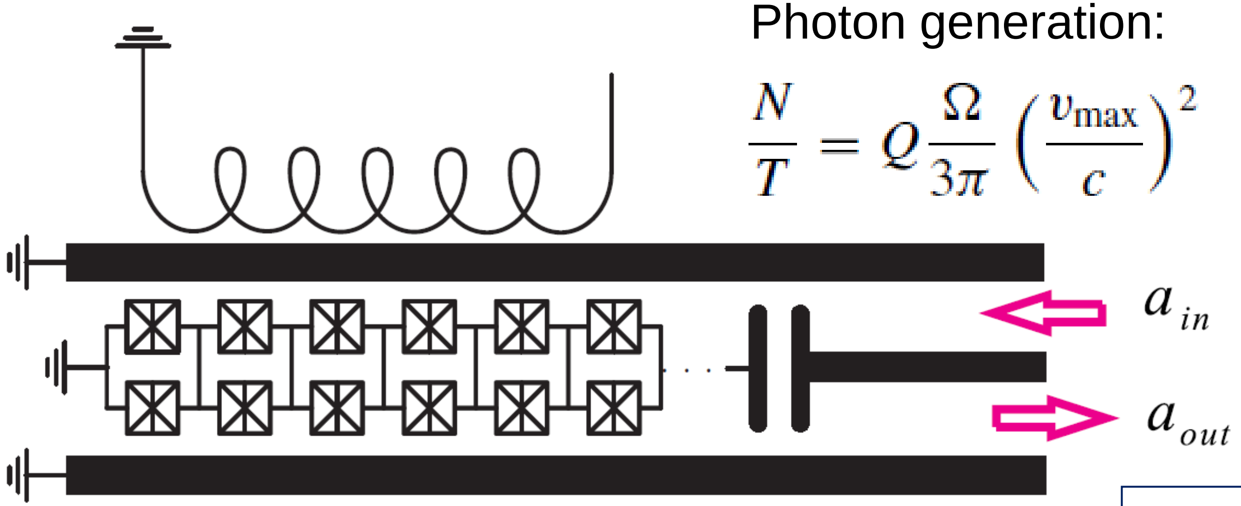
# Analogy of dynamic Casimir effect (DCE)

Photon generation:

$$\frac{N}{T} = Q\frac{\Omega}{3\pi}\left(\frac{v_{\max}}{c}\right)^2$$

Propagation speed:

$$v = \frac{1}{\sqrt{lc}}$$




$$a_{\rm out}(\omega) = e^{i\psi_\omega} a_{\rm in}(\omega)$$

$$\psi_\omega = \arctan\frac{\kappa(\omega - \omega_{\rm res})}{(\kappa/2)^2 - (\omega - \omega_{\rm res})^2}$$


G. T. Moore, J. Math. Phys. (N.Y.) 1970
E. Yablonovitch, PRL 1989
V. Dodonov, PRA 1993

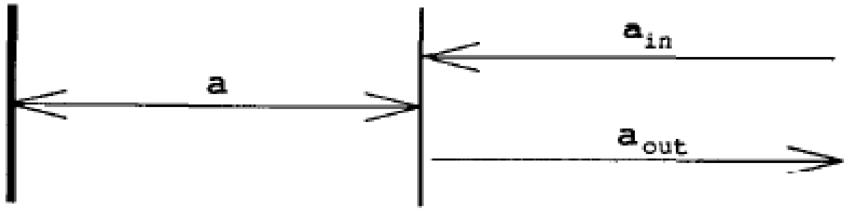
J. Johansson et al., PRL 2009, PRA 2010
C. Wilson et al., Nature 2011
P. Lähteenmäki et al., arXiv 2011

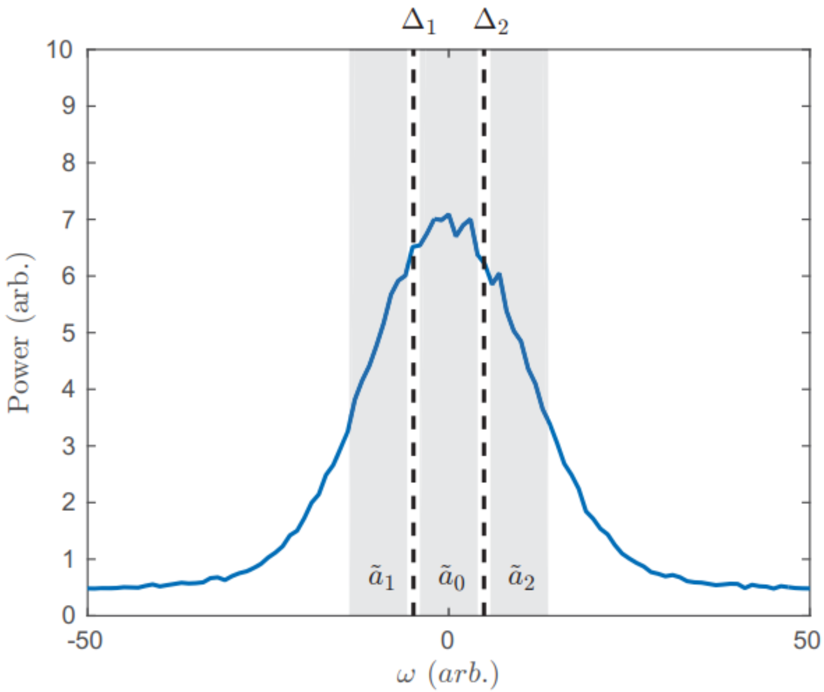
# Semiclassical theory

$$H = \hbar\omega_{res} a^\dagger a + \frac{\hbar}{2i}\sum_{p=1,2}\left[\alpha_p^* e^{i\omega_p t} - \alpha_p e^{-i\omega_p t}\right]\left(a + a^\dagger\right)^2$$

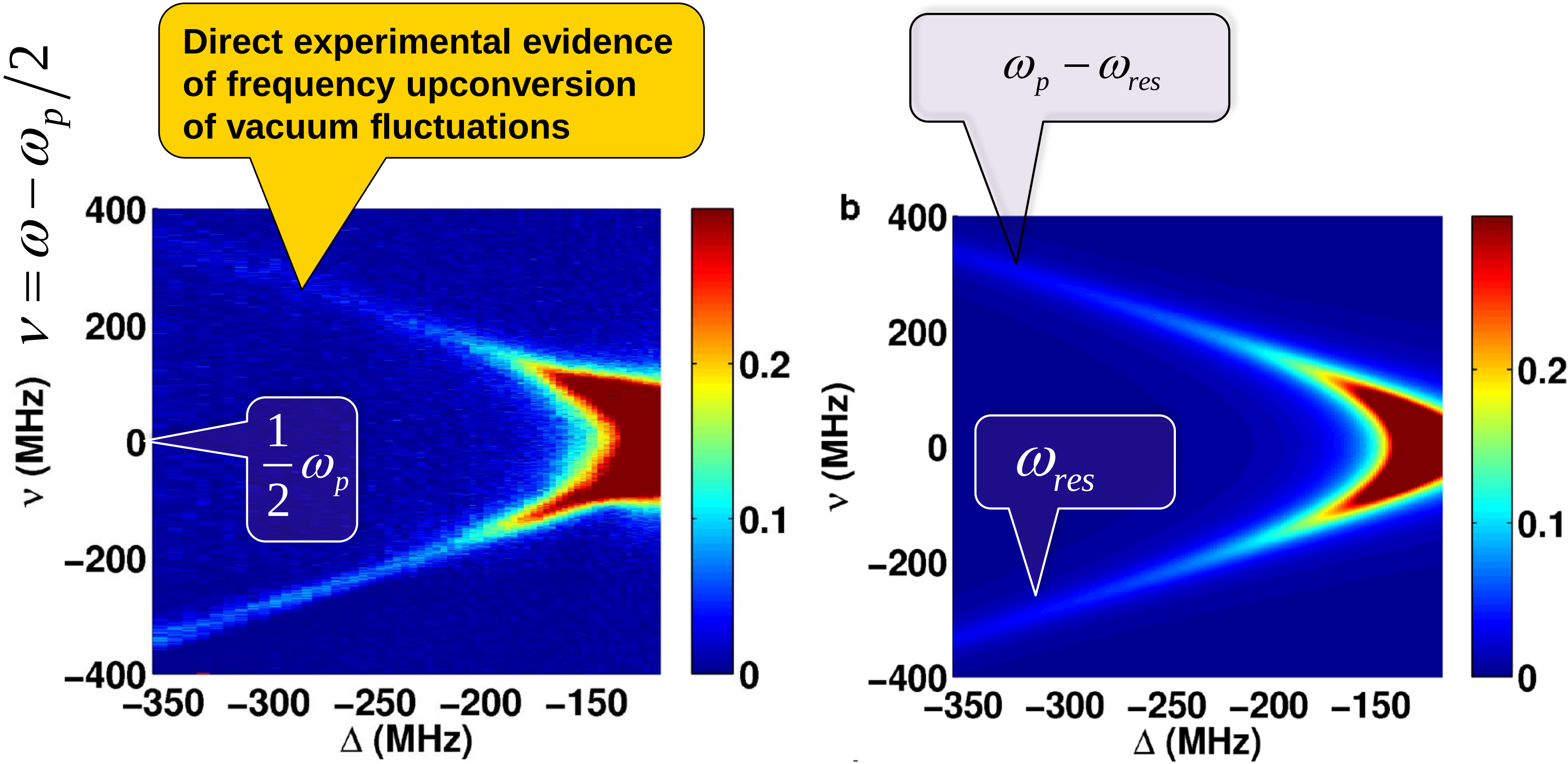
$$a(t) = \tilde{a}(t)\exp[-\omega_{res} t]$$
$$\Delta_p = \omega_p/2 - \omega_{res}$$

$$\dot{\tilde{a}} = \sum_{p=1,2} \alpha_p e^{-2i\Delta_p t}\tilde{a}^\dagger - \frac{\kappa}{2}\tilde{a} - \sqrt{\kappa}\tilde{a}_{\mathrm{in}}$$



# Measurements at large detunings




$$\Delta = \omega_{res} - \omega_p/2$$

$$\left\langle \tilde{a}^{\dagger}_{\text{out}} \tilde{a}_{\text{out}} \right\rangle$$

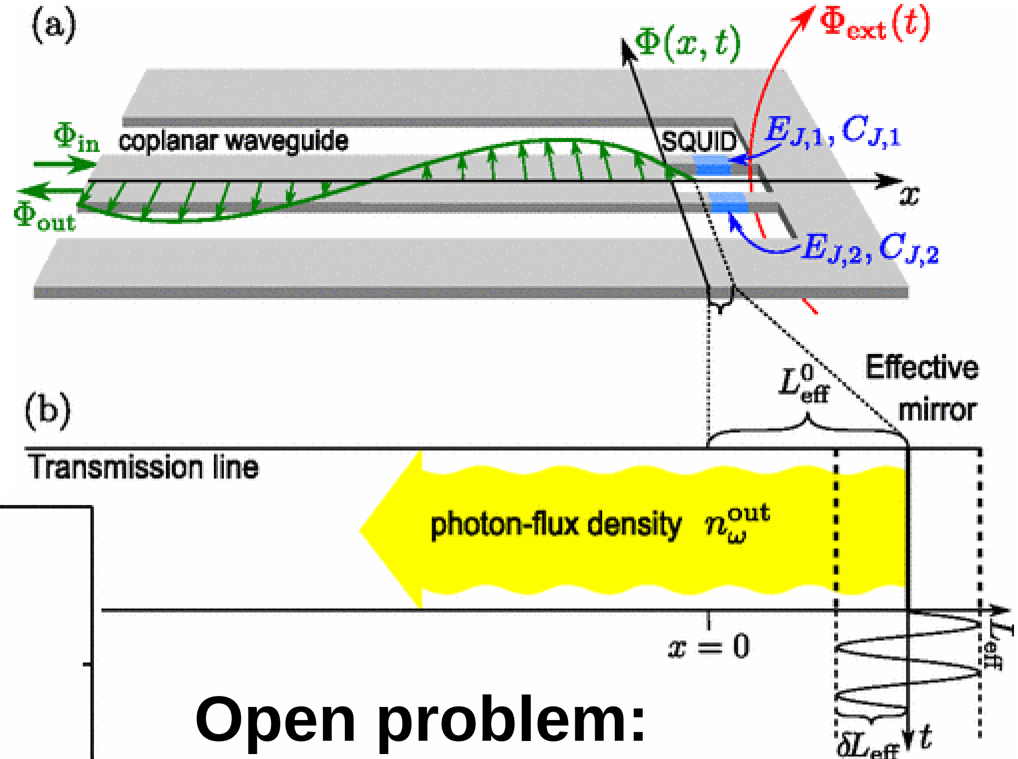


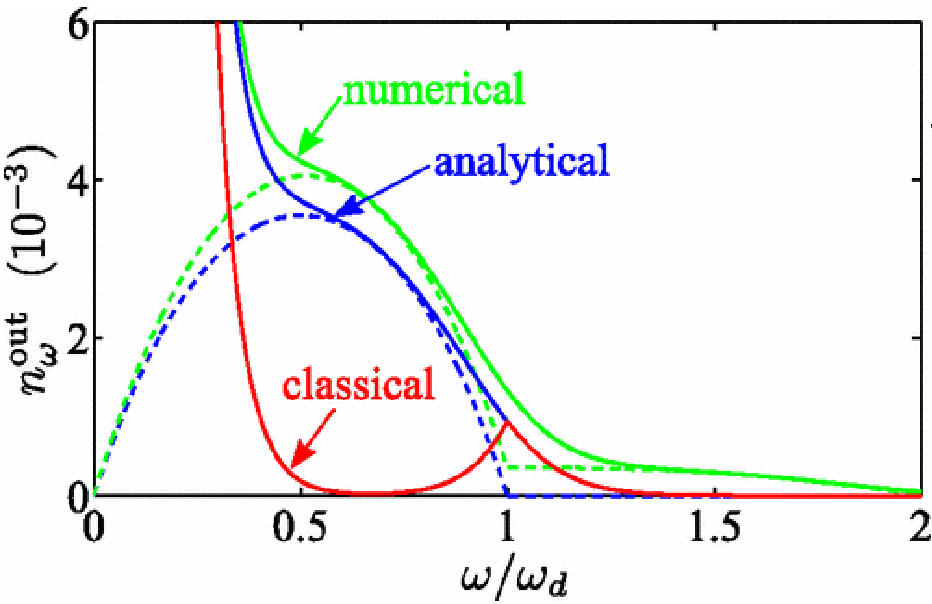
Lähteenmäki, P., Paraoanu, G. S., Hassel, J. & Hakonen, P. J. Proc. Natl. Acad. Sci. **110**, 4234–4238 (2013).

# Intrinsic spectrum of DCE

**Need a semi-infinite transmission line**
- better sensitivity
- broad band

C. Wilson et al., Nature 2011




**Open problem:**
Specific parabolic spectrum expected

J. R. Johansson et al., PRL 103, 147003 (2009)

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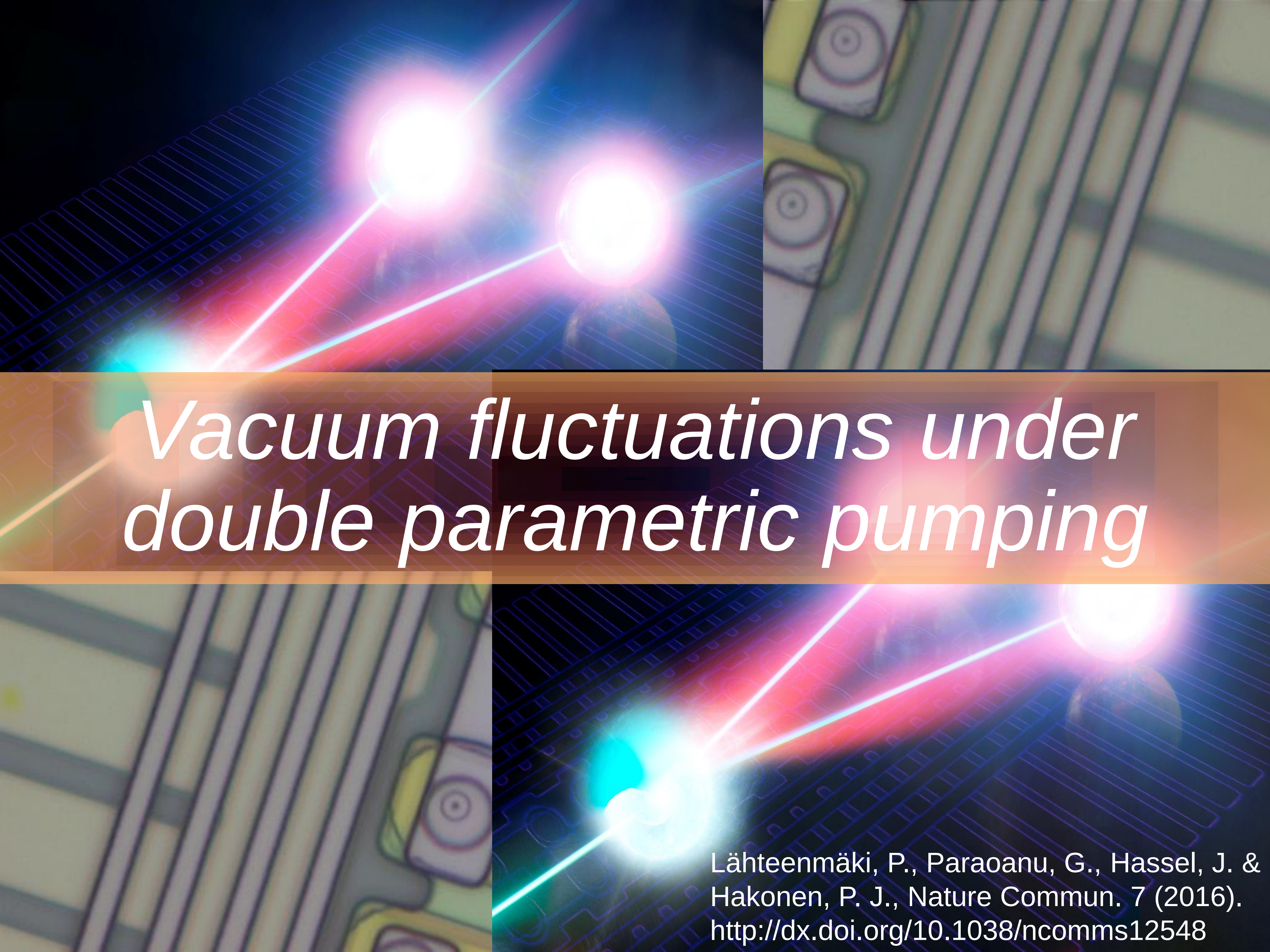
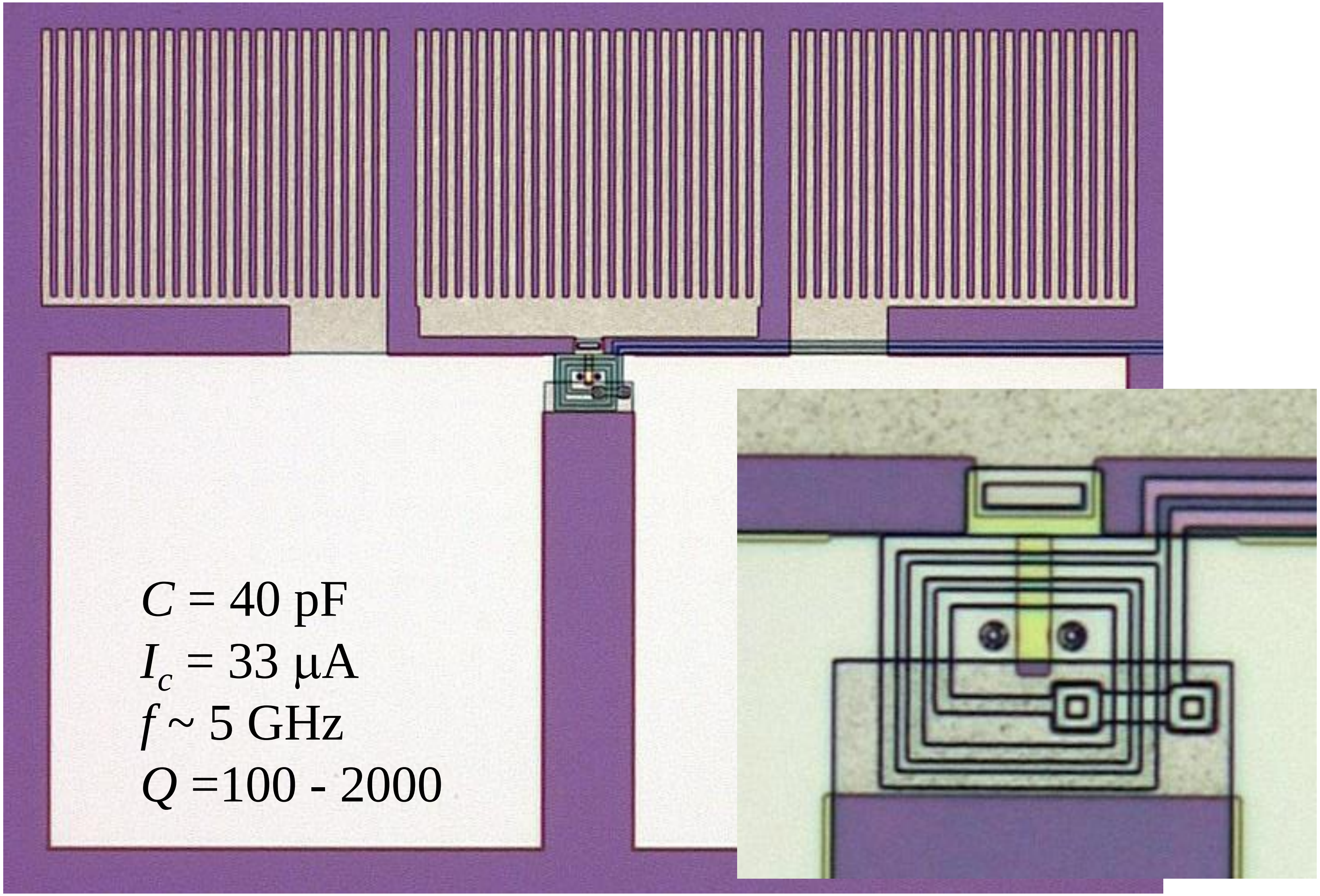
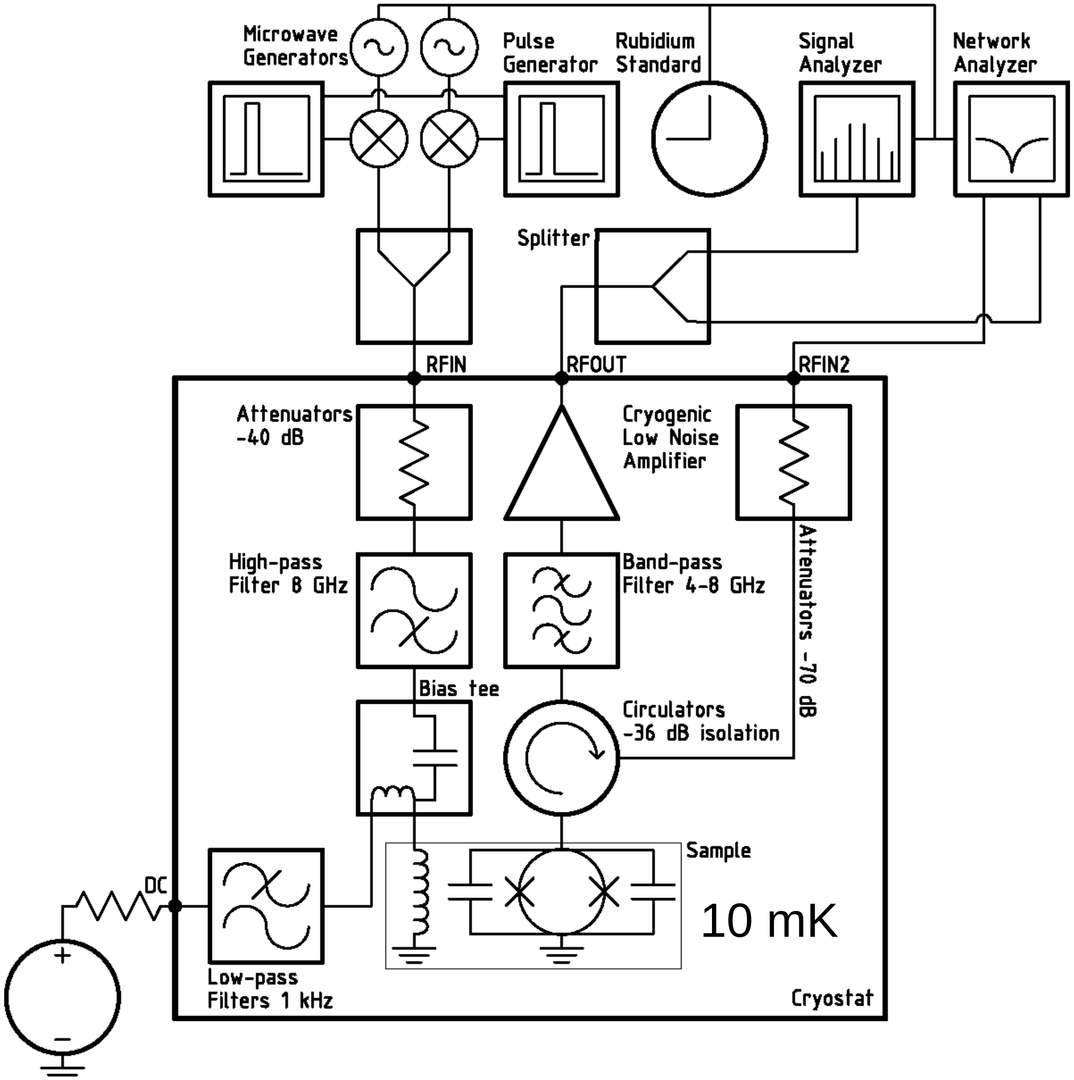

Vacuum fluctuations under double parametric pumping
Lähteenmäki, P., Paraoanu, G., Hassel, J. & Hakonen, P. J., Nature Commun. 7 (2016).
http://dx.doi.org/10.1038/ncomms12548

# Lumped element parametric device



# Experimental setup

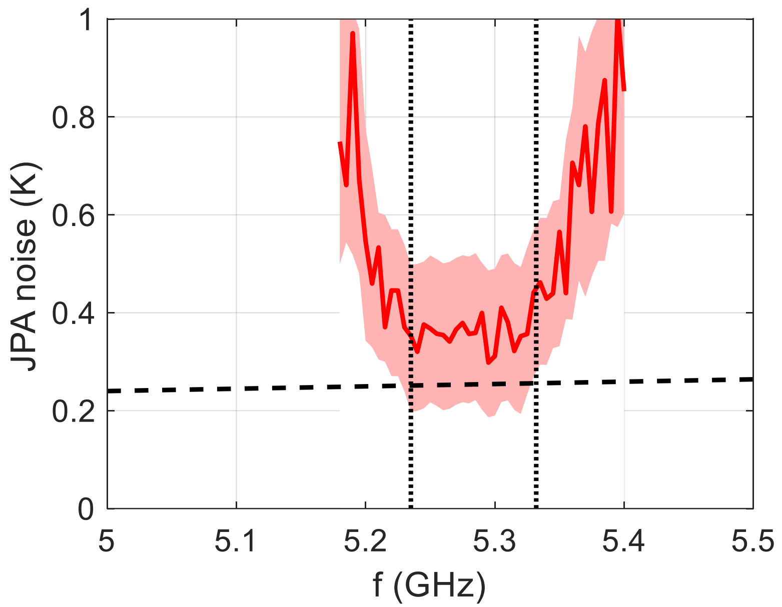


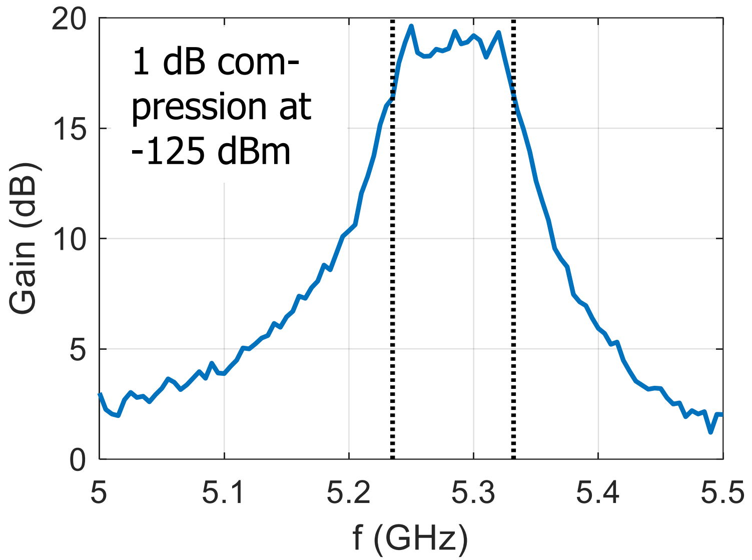

- Vector signal analyzer

- Quadrature components digitized at 50 MHz rate

- Digital band filtering and correlations with FFT

# Gain and Noise Performance




- ❑ JPA noise using SNR improvement
  - ○ Vertical lines: 3 dB bandwidth
  - ○ Dashed line: quantum limit
  - ○ Shaded area: measurement error

T. Elo, et al. 2018

- ❑ 100 MHz bandwidth
- ❑ Approching quantum limit

$$T_{QN} = \frac{\hbar\omega}{k_b}$$

- ❑ Observation of quantum squeezing indicates low losses

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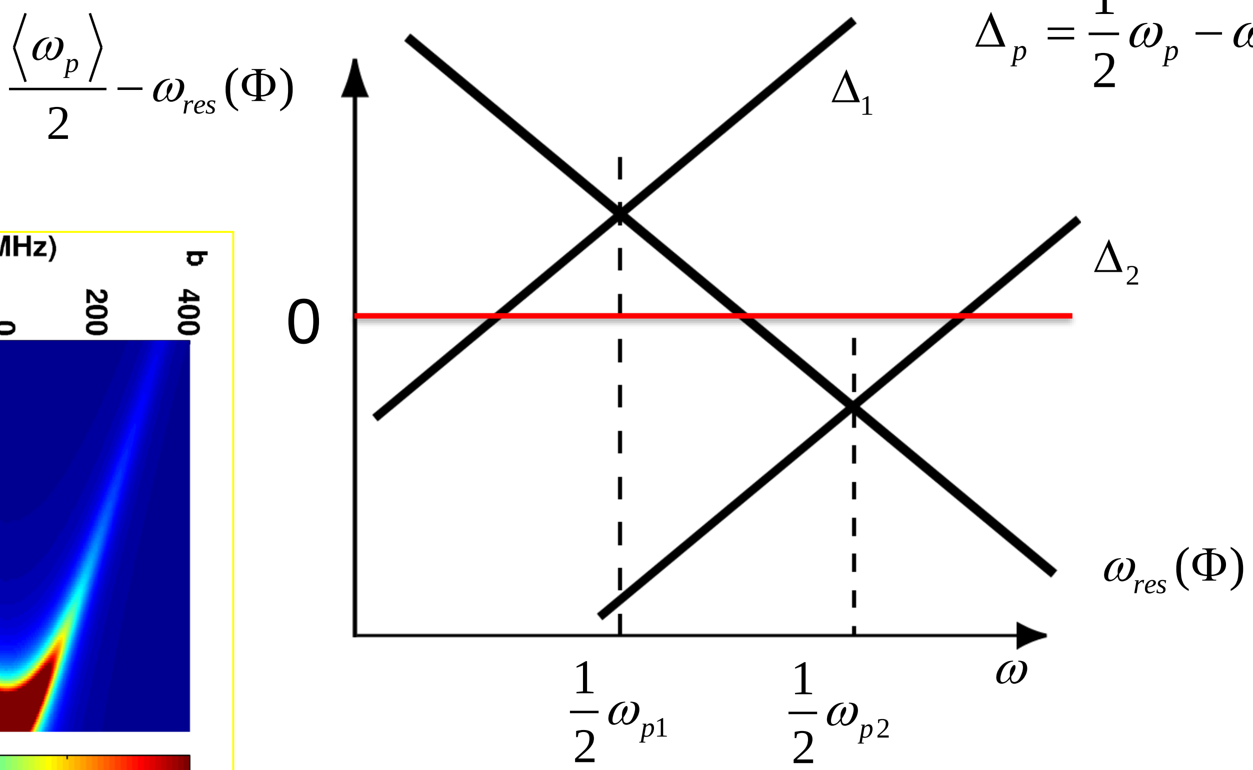
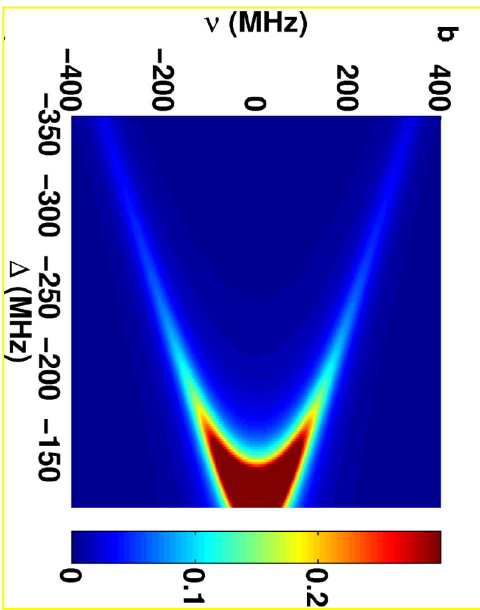
# Correlations in a two pump configuration

$$H = \hbar\omega_{res}a^{\dagger}a + \frac{\hbar}{2i}\sum_{p=1,2}\left[\alpha_p^{*}e^{i\omega_p t} - \alpha_p e^{-i\omega_p t}\right]\left(a + a^{\dagger}\right)^2$$

$$\Delta_p = \frac{1}{2}\omega_p - \omega_{res}$$

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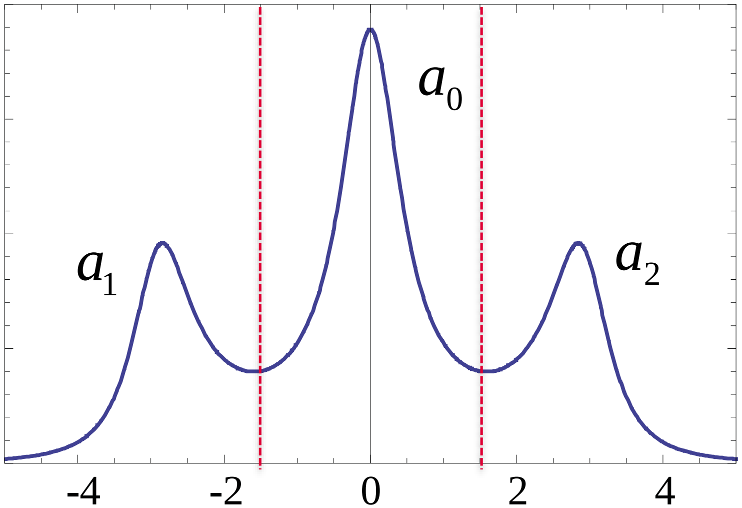
# Bright and dark modes

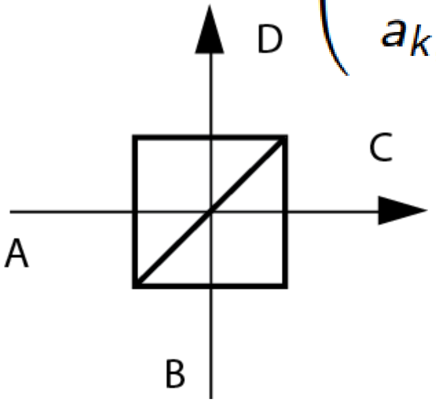



$$\left\{ \begin{array}{l} \langle \tilde{a}_{out} \tilde{d}_{out} \rangle \\ \langle \tilde{b}_{out} \tilde{d}_{out} \rangle \\ \langle \tilde{d}^{\dagger}_{out} \tilde{d}_{out} \rangle \end{array} \right\} = 0$$

$$\begin{pmatrix} a_{k,C} \\ a_{k,D} \end{pmatrix} = U \begin{pmatrix} a_{k,A} \\ a_{k,B} \end{pmatrix}$$

*Bright state* $$\tilde{b} = \frac{1}{\sqrt{2}}(\tilde{a}_1 + \tilde{a}_2)$$

*Dark state* $$\tilde{d} = \frac{1}{\sqrt{2}}(\tilde{a}_1 - \tilde{a}_2)$$

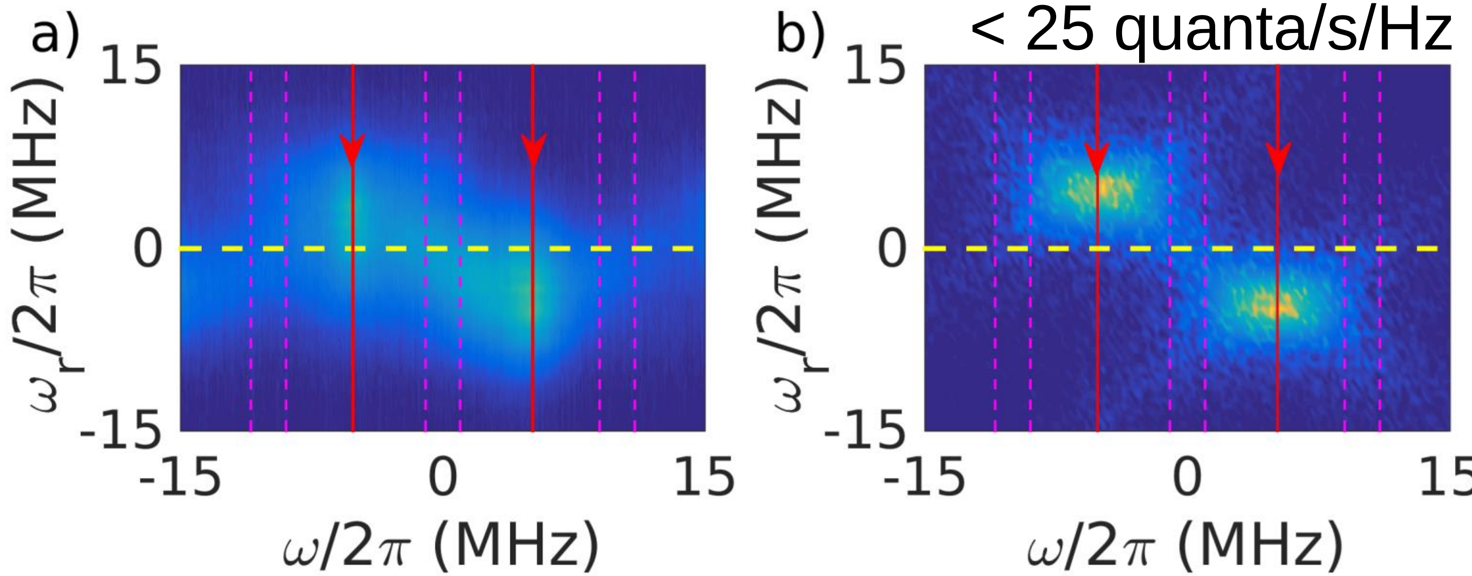


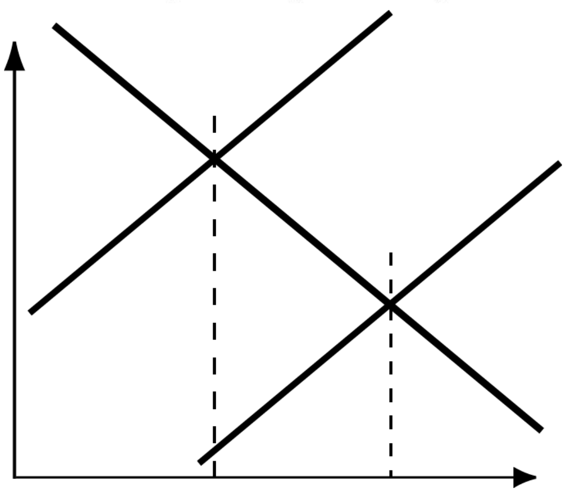

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

**- Coherence due to the same quantum fluctuation taking part in the generation of the pairs**

# Noise power measurements (low power)

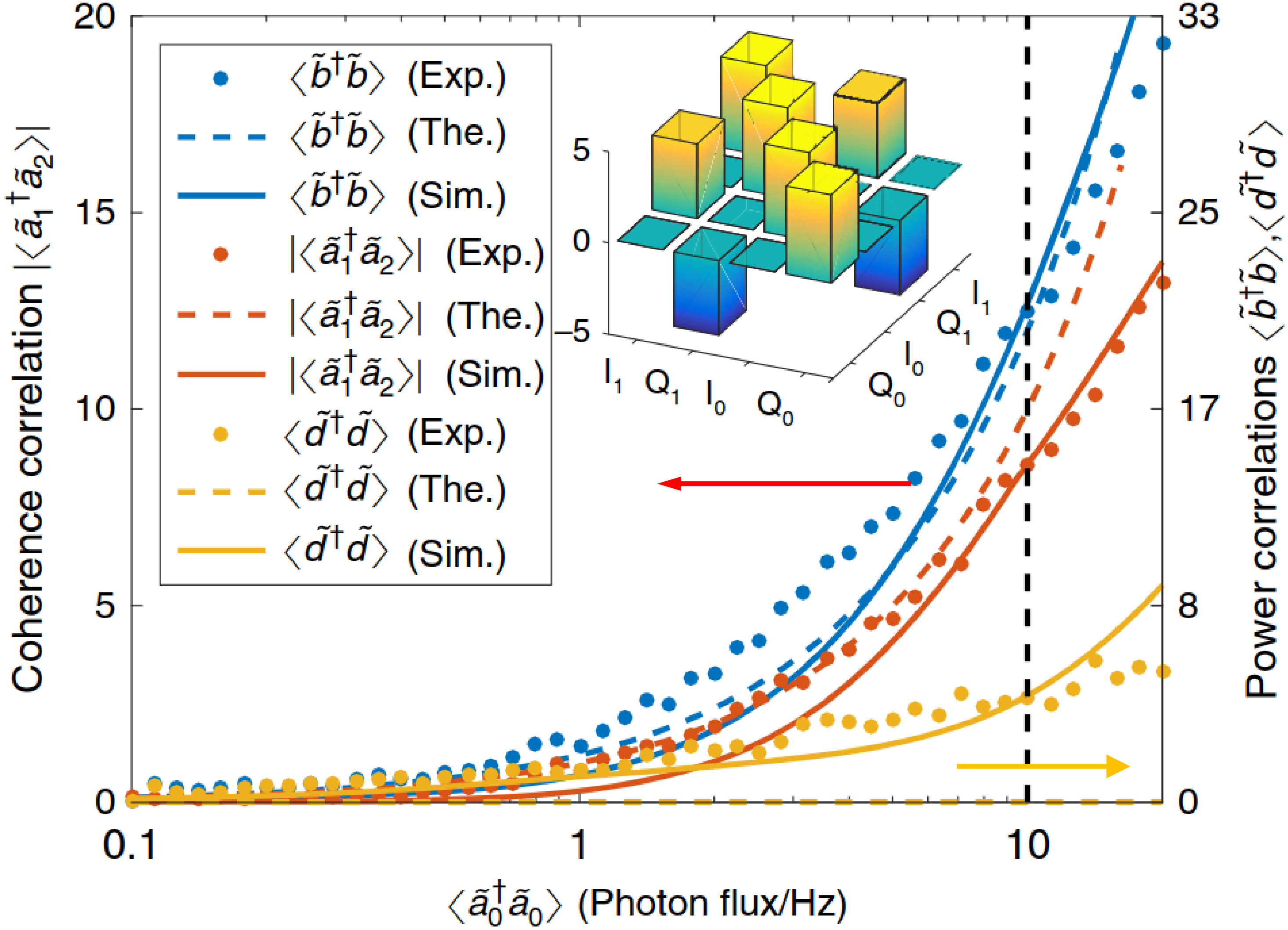
< 25 quanta/s/Hz $\omega_r = \omega_{res}$



$$\dot{\tilde{a}} = \sum_{p=1,2} \alpha_p e^{-2i\Delta_p t}\tilde{a}^\dagger - \frac{\kappa}{2}\tilde{a} - \sqrt{\kappa}\tilde{a}_{\mathrm{in}}$$

# Mode correlators

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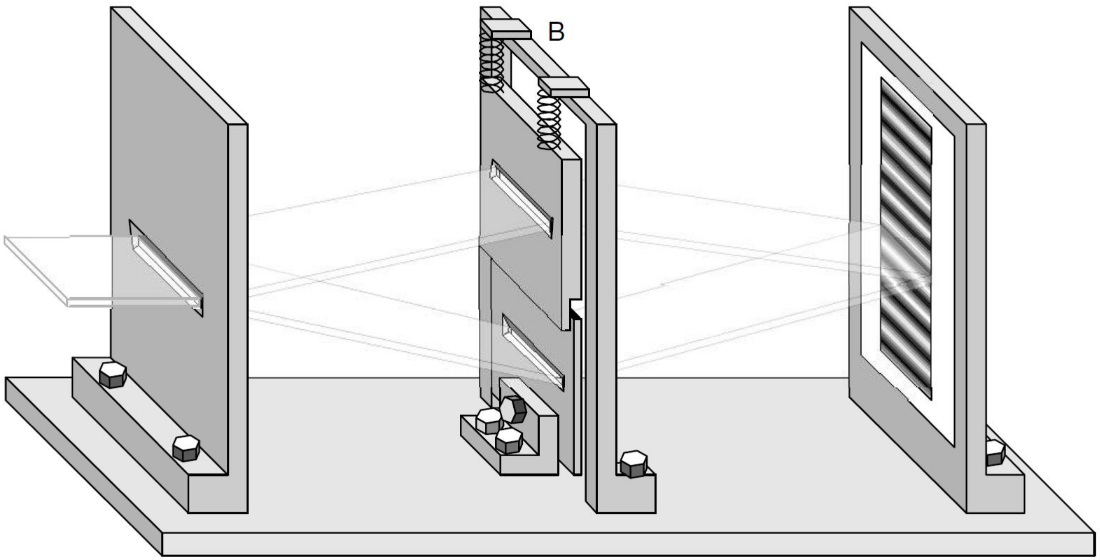
Lähteenmäki, P., Paraoanu, G. S., Hassel, J. & Hakonen, P. J.,
Nature Communications **7** (2016). http://dx.doi.org/10.1038/ncomms12548

# Which path - which color information

In our case: two **slits open when two pumps are on** – the system does not know from which pump the photon came




- Our which path is in **frequency space**
- Information can be obtained by varying pumps in time

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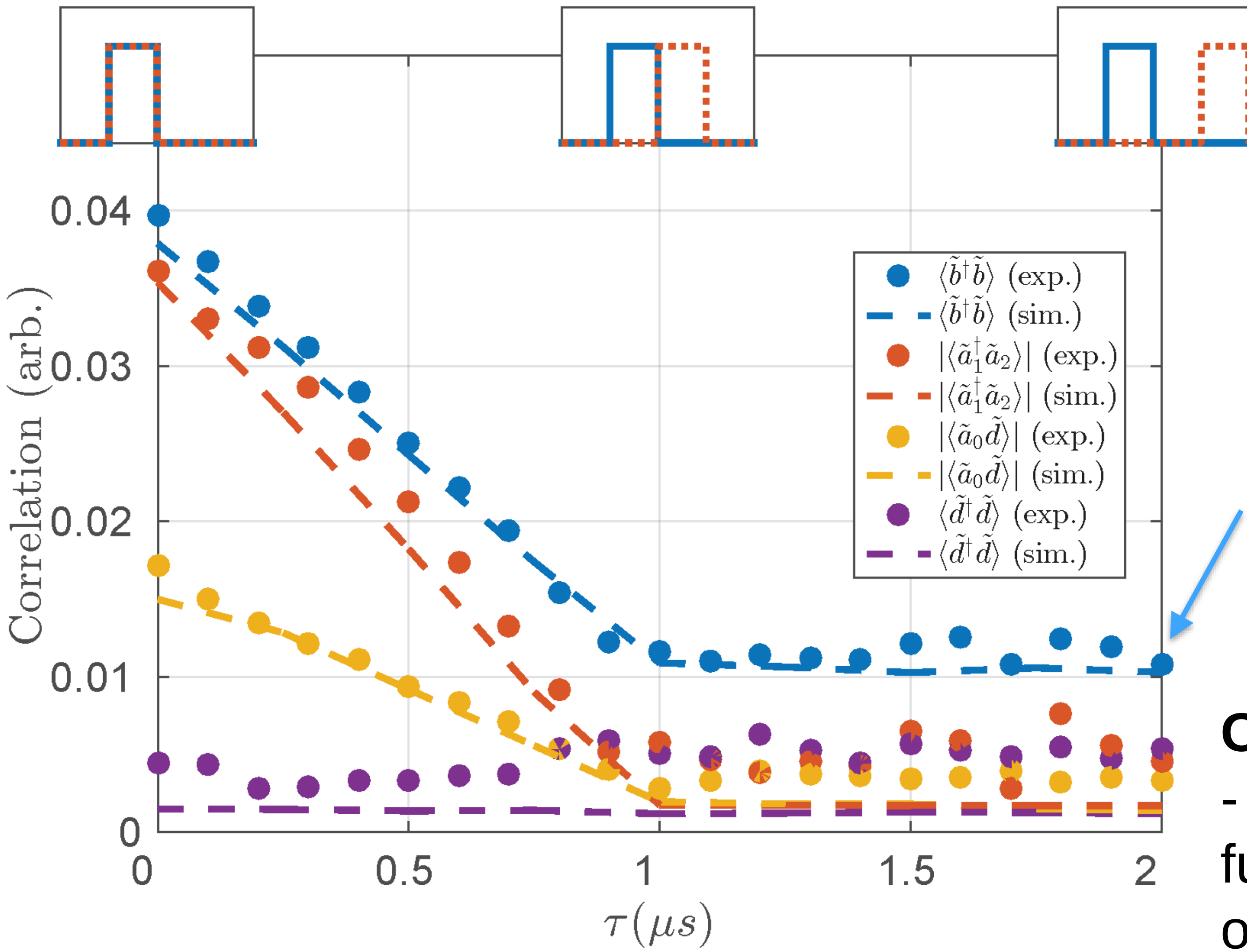
P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. M. Raimond & S. Haroche, Nature **411**, 166 (2001)

# Pulsed pumps with tuned overlap




$\kappa_I \approx 1\,\mathrm{MHz}$
$\kappa_E \approx 2\,\mathrm{MHz}$
$Q \approx 1700$

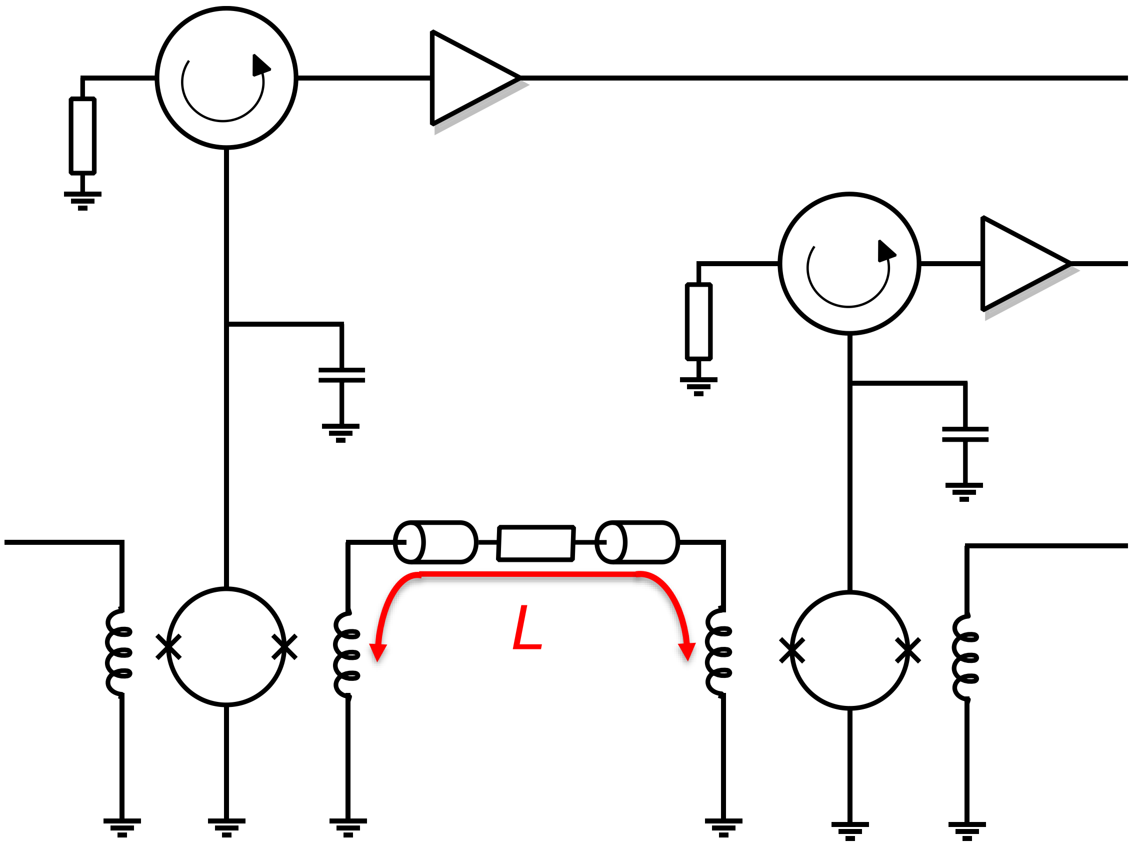
Base line for $b^\dagger b$ contains DCE power

**Open problem:**

- How correlation functions depend on time

# Vacuum induced coherence: open questions

1) Correlations with increased separation *L* of the loops
2) Past – future correlations/The Unruh effect




Quantum vacuum provides non-local correlations?

Pumps on for a time $T < L/c$
➡ Correlations? Decay with *L*?

# The vacuum and relativity


## Minkowski metric

$$(\Delta s)^2 = (\Delta ct)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

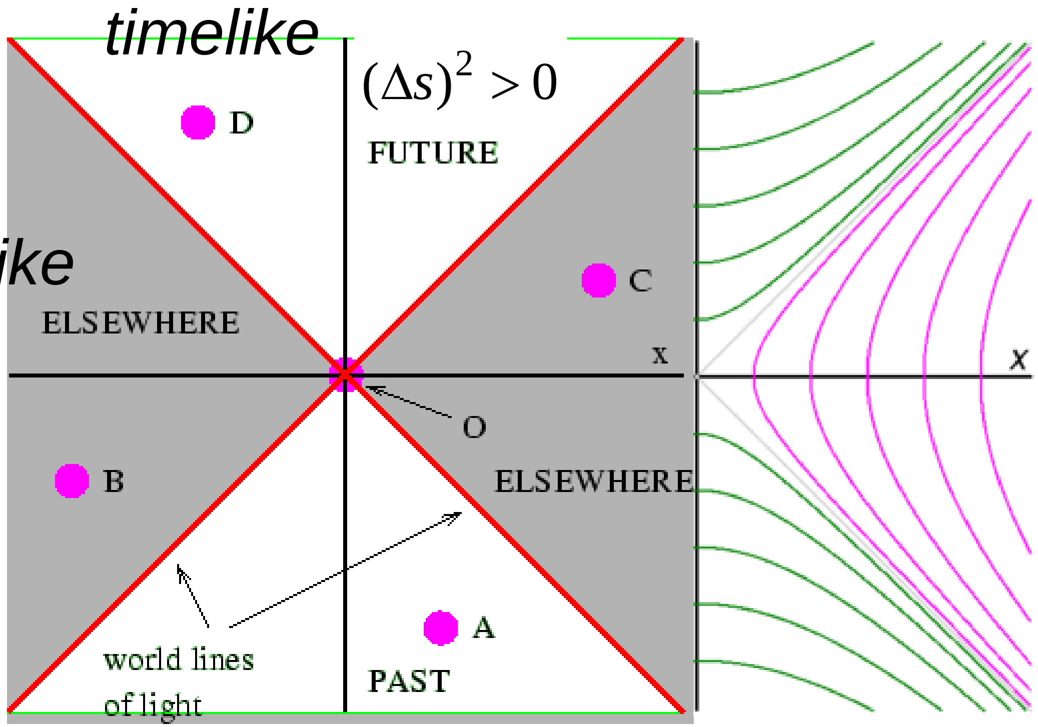
- The past light cone contains all the events that could have a causal influence on O

$x = \pm ct$

*spacelike* region

$(\Delta s)^2 < 0$

**Light cone**




**Non-local correlations** via quantum vacuum*

Also **past-future correlations**

Closely related to the **Unruh effect**

*A. Valentini, Phys. Lett. A **153**, 321 (1991)
*B. Reznik, et al., Phys. Rev. A **71**, 042104 (2005)

# The vacuum and the equivalence principle

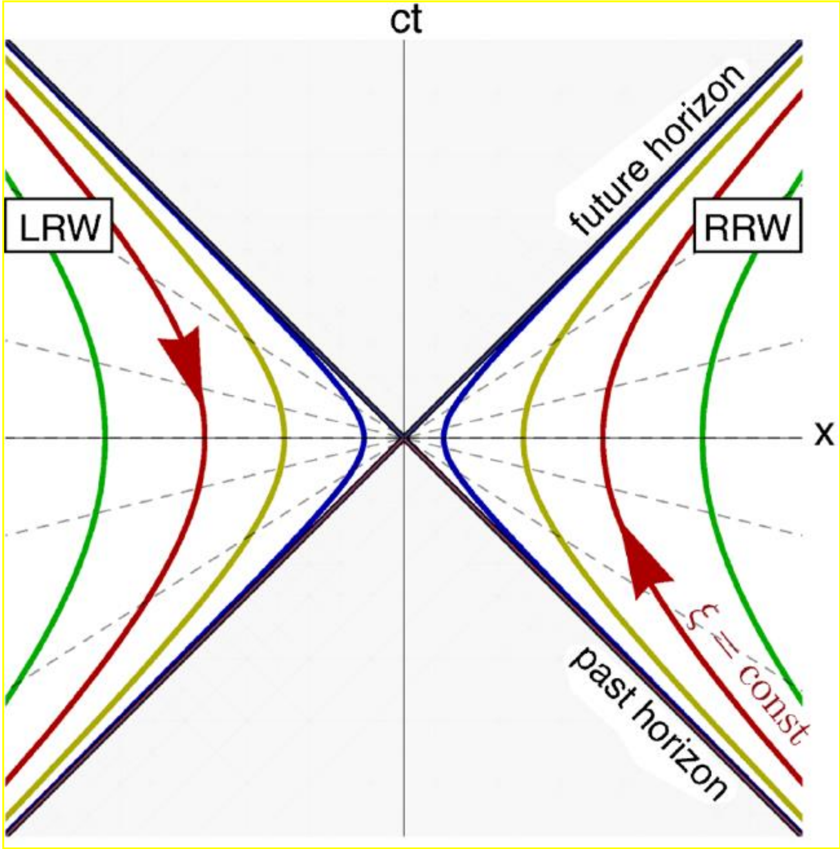
## The Unruh effect:

- An accelerating observer will observe **blackbody radiation** where an inertial observer would observe none.

- The uniformly accelerating observer is out of causal contact with part of space time (having both positive and negative *f*)

$$k_B T_U = \frac{\hbar a}{2\pi c}$$

How to observe?

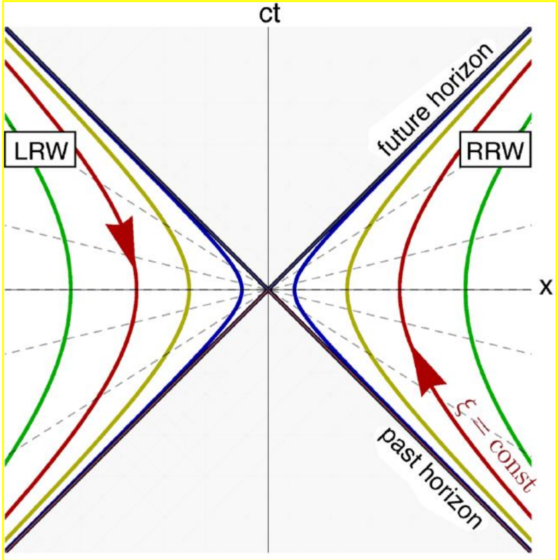


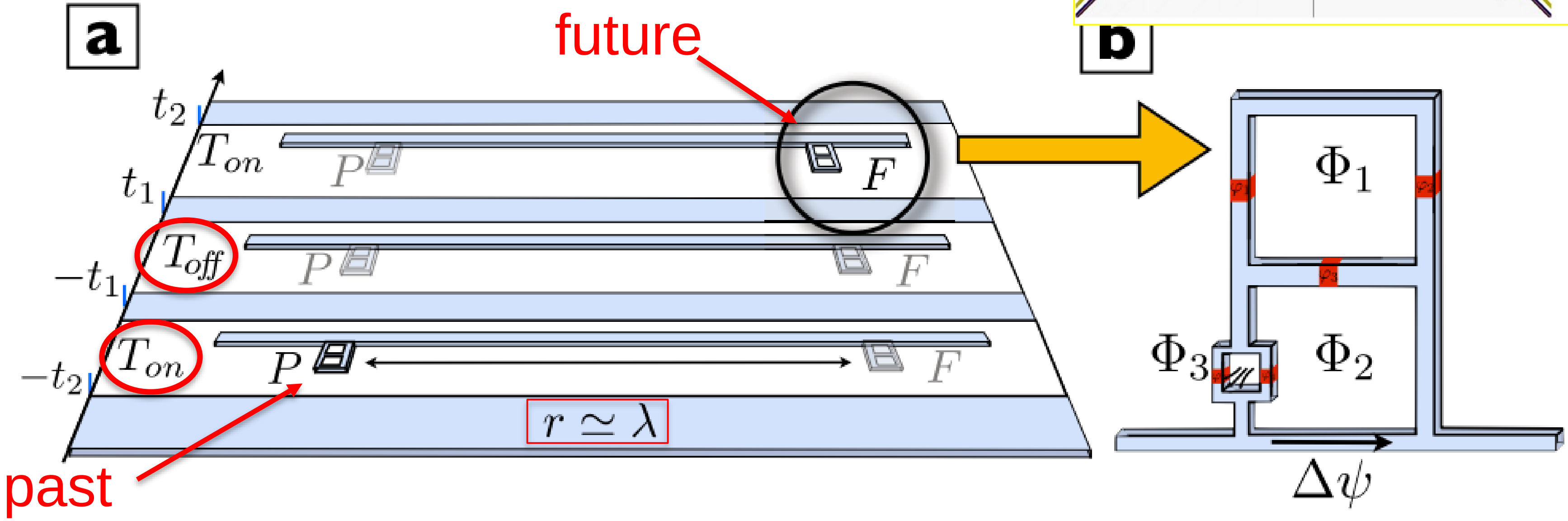

$$T_U = 1.2\times10^{-19} K \quad a = 10\, m/s^2$$

S. Fulling 1973, P. Davies 1975, and W. G. Unruh 1976

# Past-Future Vacuum Correlations in Circuit QED

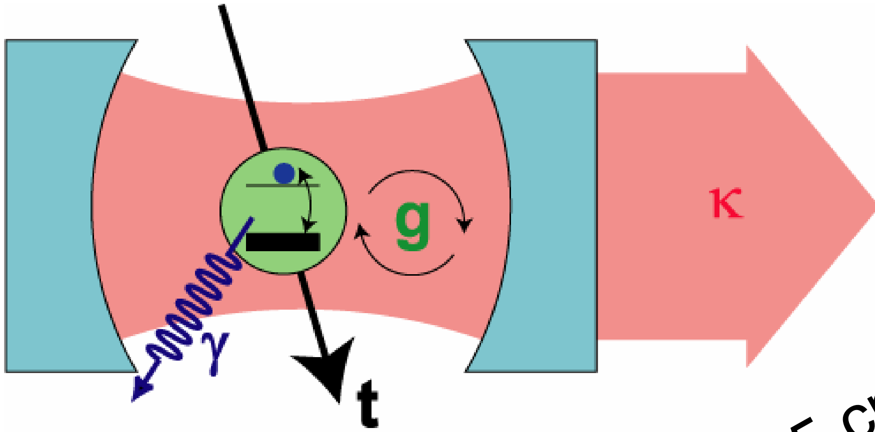
- Entanglement across O
- Qubits like Unruh de Witt detectors:
  operate detectors with ***varying splitting***
  instead of acceleration or ***at different t***
- Small level spacing -> long time scales
- Coherence times sufficient




S. J. Olson and T. C. Ralph, Phys. Rev. A **85**, 012306 (2012)
C. Sabin, et al. PRL **109**, 033602 (2012)

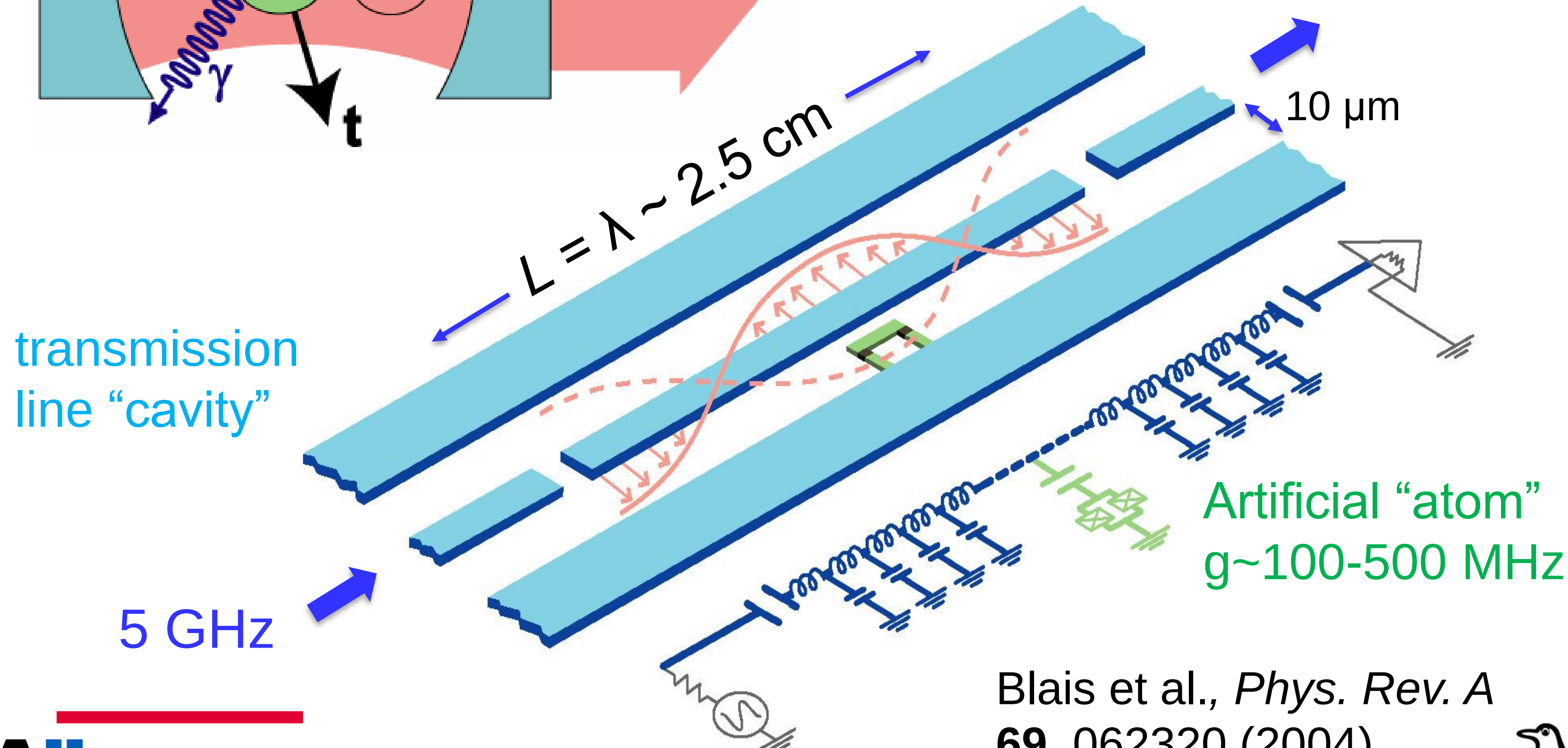
# Circuit QED




2g = vacuum Rabi frequency

κ = cavity decay rate

γ = “transverse” decay rate




Blais et al., *Phys. Rev. A* **69**, 062320 (2004)

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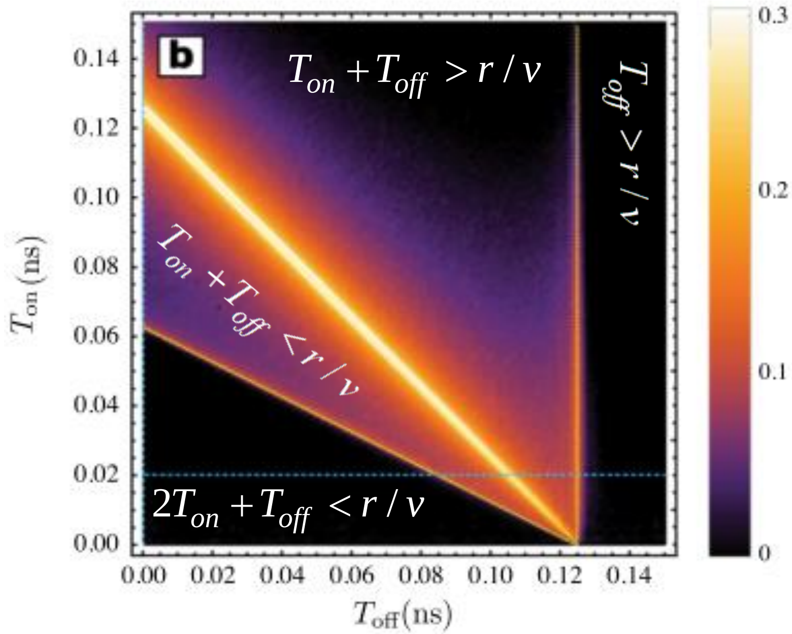
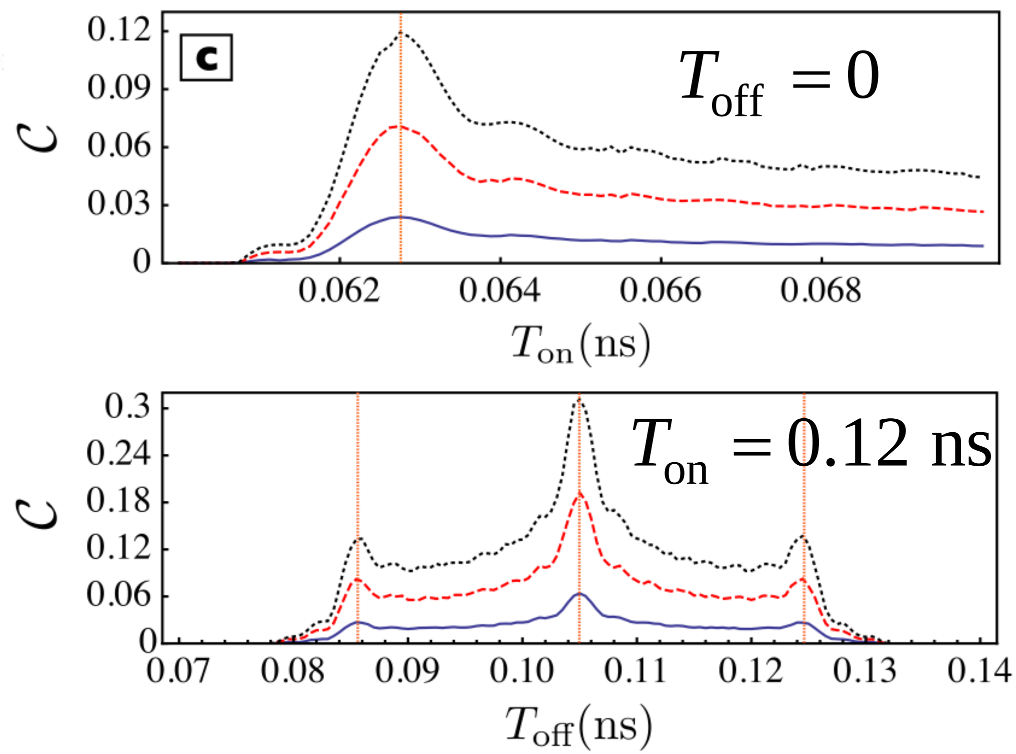
# Past-Future Vacuum Correlations in Circuit QED




$\Omega_P = \Omega_F = 2\pi \times 1\text{GHz}$ Qubit splitting

$g/\Omega = 0.19$ Qubit coupling

$r/\lambda = 0.125$ Scaled distance




Challenges:
- Low electronic T
- Fast pulsing
- Rapid low-noise measurement

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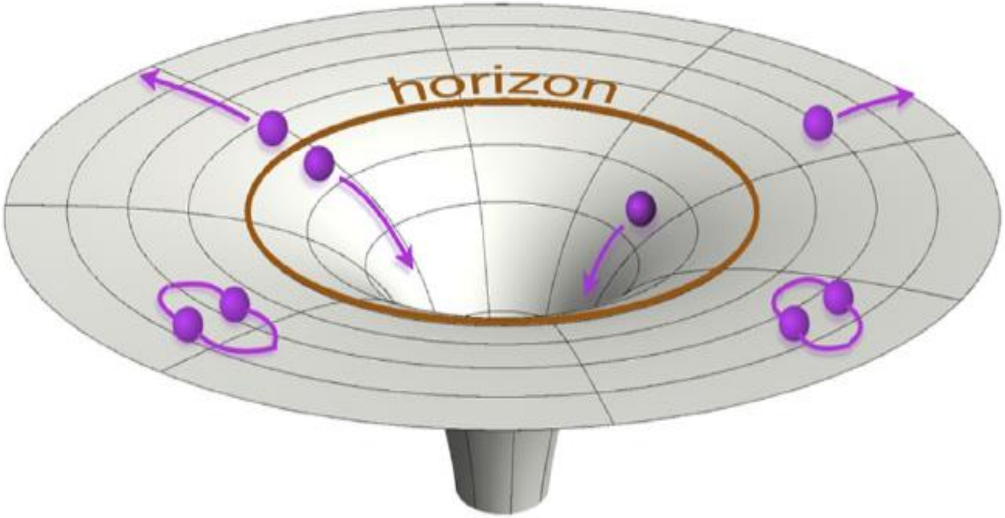
C. Sabin, et al. PRL **109**, 033602 (2012)

# Gravitational effects and its analogs

## The Hawking effect

(1974)

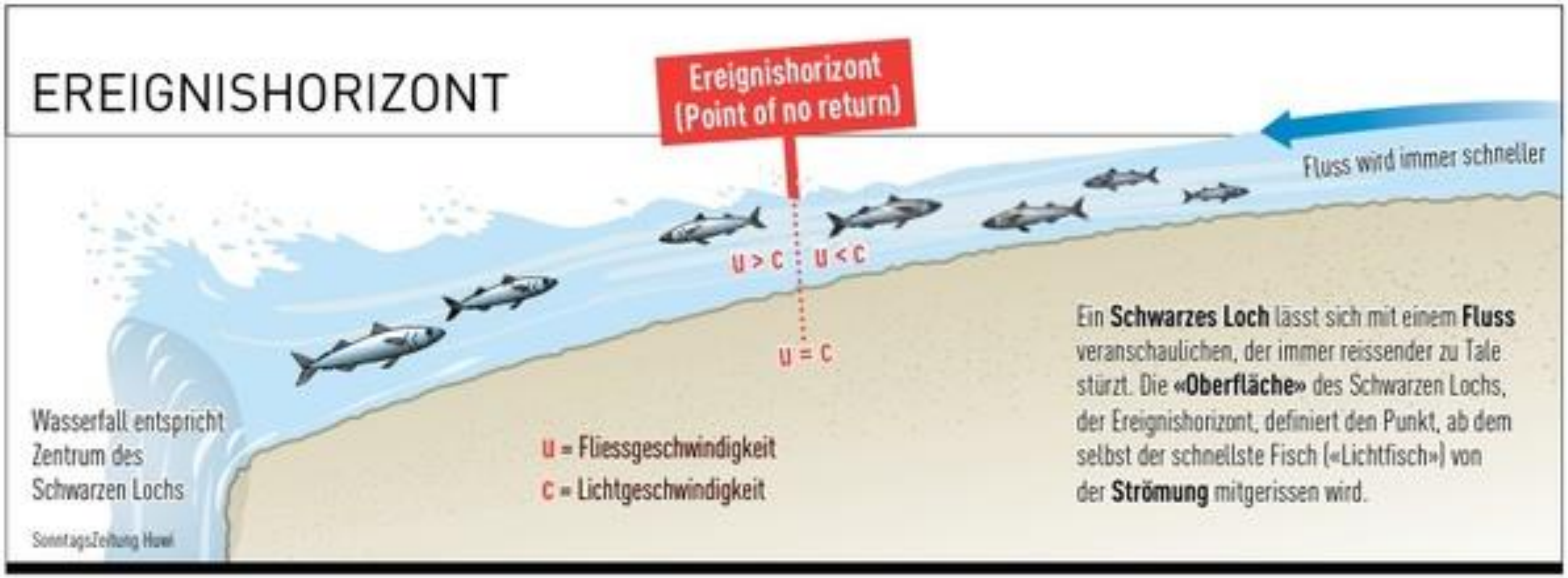
$$k_B T_H = \frac{\hbar g_h}{2\pi c} \qquad g_h = \frac{c^4}{4GM}$$


**Estimate:**

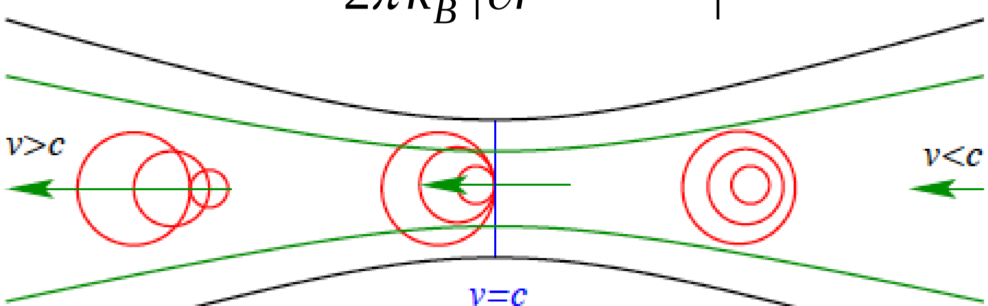
**For a black hole with *M* = the solar mass**
$T_H = 10^{-7} K$ **... but the c.m.b. is at 2.7 K**

# Sonic analog of black holes




$$T_{\text{Hawking}} = \frac{\hbar}{2\pi k_B}\left|\frac{\partial}{\partial r}\left(v_0 - c\right)\right|$$

Unruh 1980




Schutzhold 2008

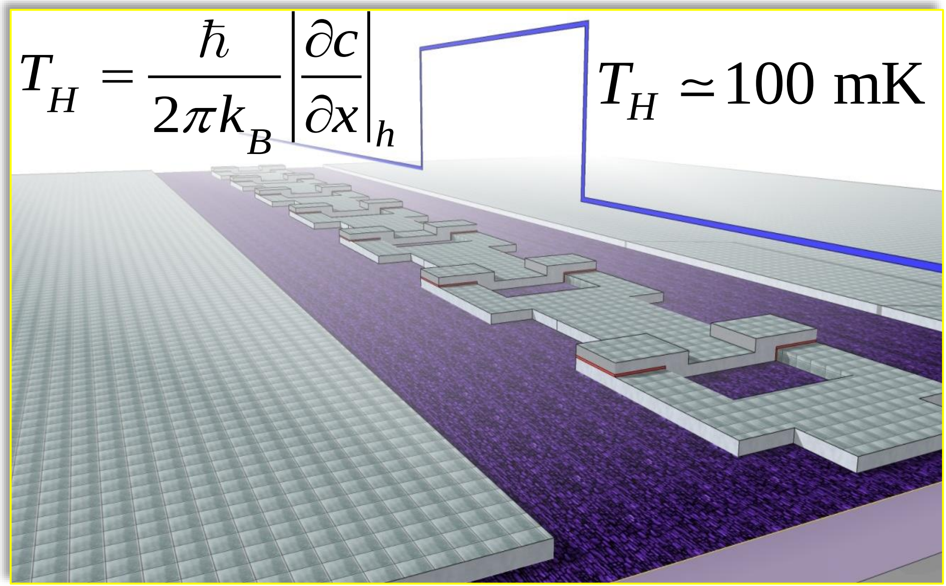
In Bose-Einstein condensates:

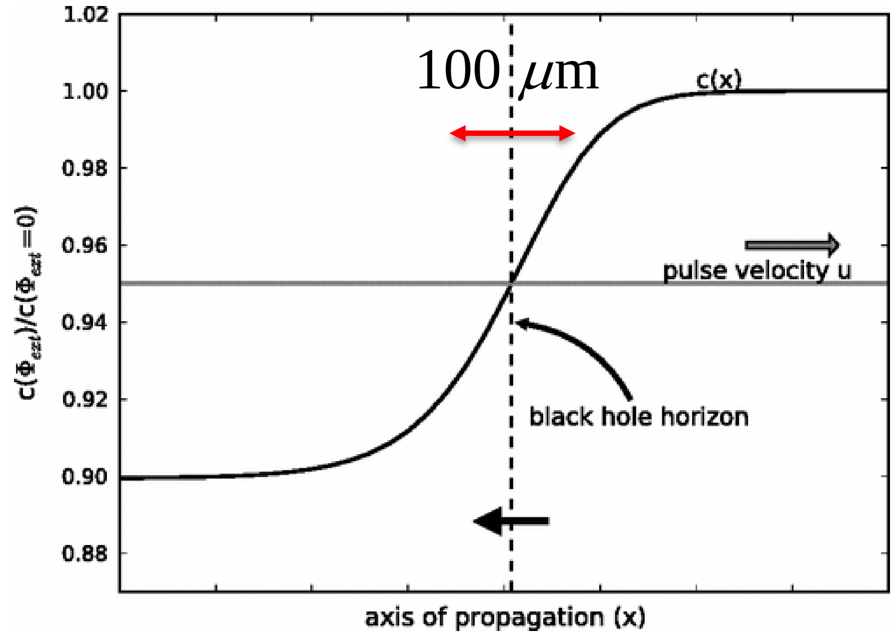
**Observation of quantum Hawking radiation and its entanglement in an analogue black hole**

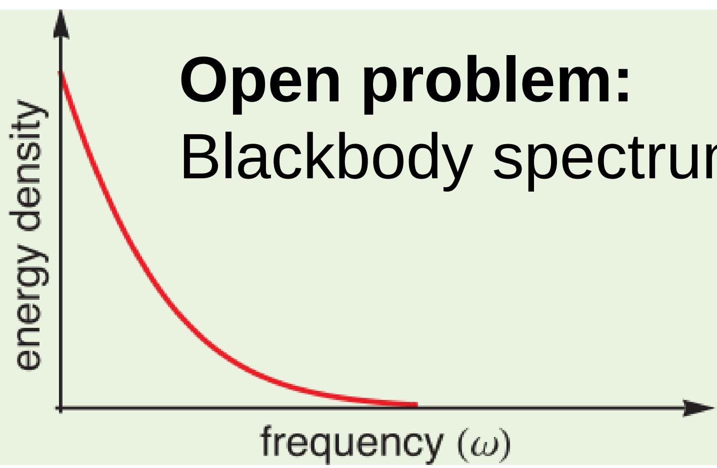
J. Steinhauer
*Nature Phys.* **12**, 959 (2016)

Aalto University

# Analog cosmological effects in SQUID arrays




**Open problem:**
Blackbody spectrum

R. Schützhold, and W. G. Unruh, Phys. Rev. Lett. **95**, 031301 (2005)
D. Nation, M. P. Blencowe, A. J. Rimberg, and E. Buks, Phys. Rev. Lett. **103**, 087004 (2009)

Blackbody spectrum in acoustic systems?
Silke Weinfurtner, et al., Phys. Rev. Lett. **106**, 021302 (2011)

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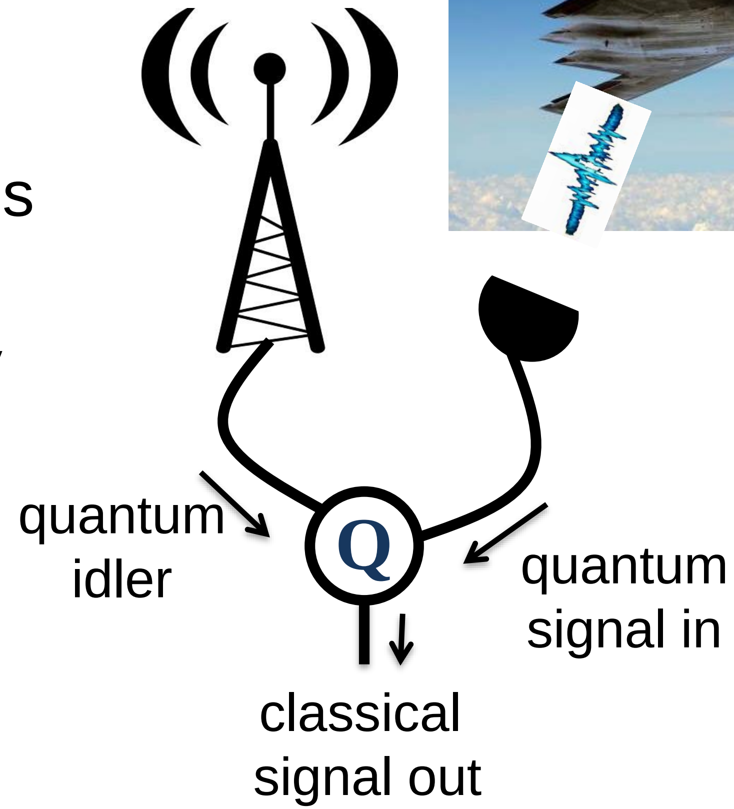
# Entanglement as a resource: quantum radar

- Quantum correlations (entanglement) shared by transmitted and idler radiation
- Receiver distills the correlations from the incoming radiation
- Particularly useful in extremely lossy and noisy situations.




→ **Higher sensitivity with less power**

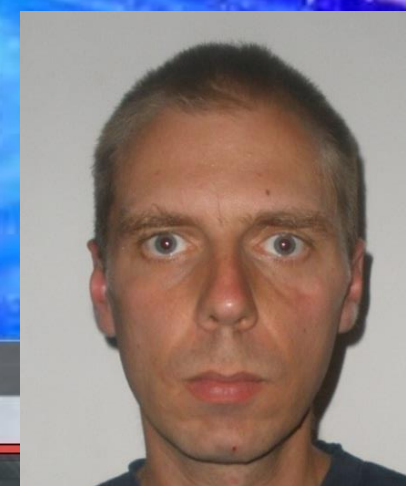
**Application: detection of stealth aircrafts**

***"China's latest quantum radar won't just track stealth bombers, but ballistic missiles in space too"***

Pasi Lähteenmäki

Sorin Paraoanu

Teemu Elo



Juha Hassel

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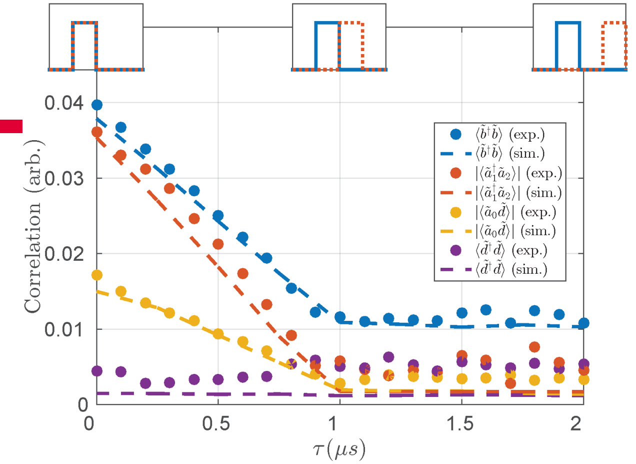
# Open problems summary

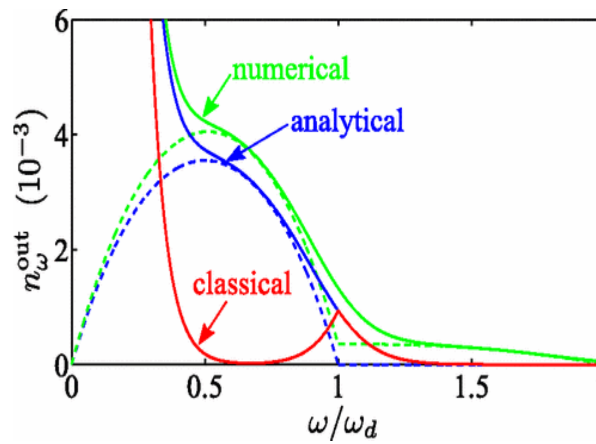
**1) *Casimir photon generation***
- *time dependent phenomena*
- *parabolic spectrum*

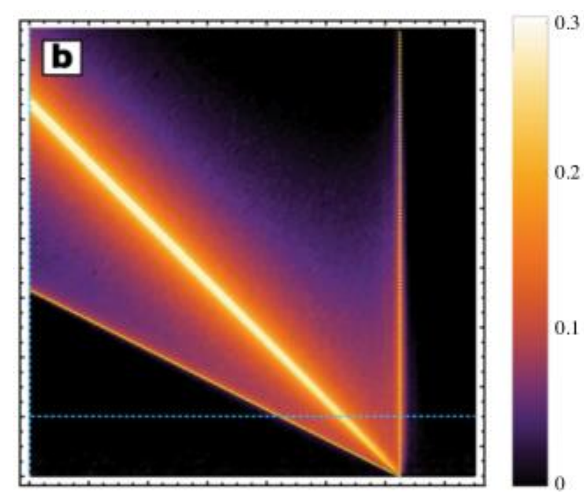
**2) *Hawking radiation***
- *analog using electronic circuits*
- *blackbody spectrum*

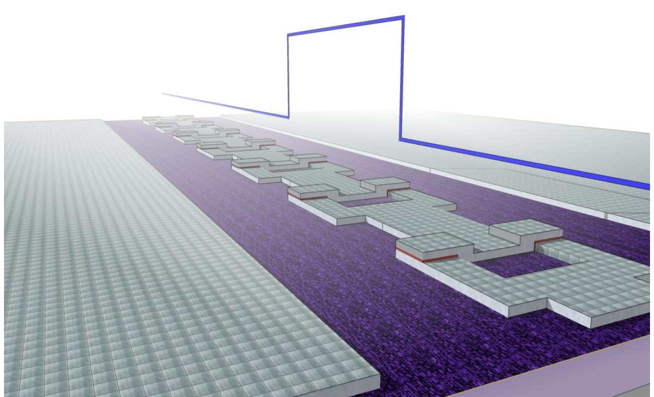
**3) *Past – future correlations***
- *entanglement transfer of quantum vacuum*
- *sub-nanosecond, low noise measurements*

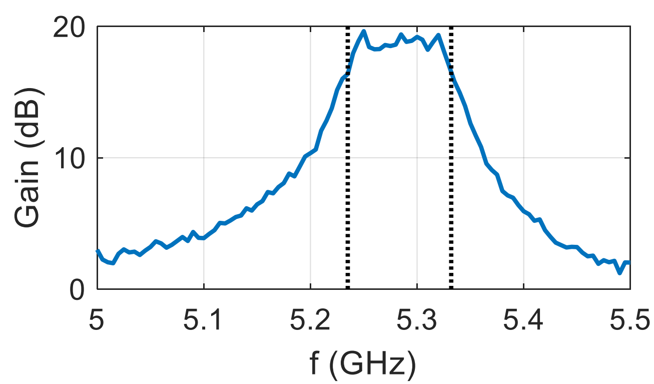
**4) *Quantum radar***
- *how to use entanglement to improve SNR*





# "Mode" observables: Quadratures

**Quadrature operators (like x and p):**

$$H_{\mathrm{k}} = \hbar\omega_{\mathrm{k}}(a^{\dagger}_{\mathrm{k}}a_{\mathrm{k}} + \frac{1}{2}))$$

$$X_1 = \frac{1}{\sqrt{2}}\left(a^{\dagger} + a\right)$$

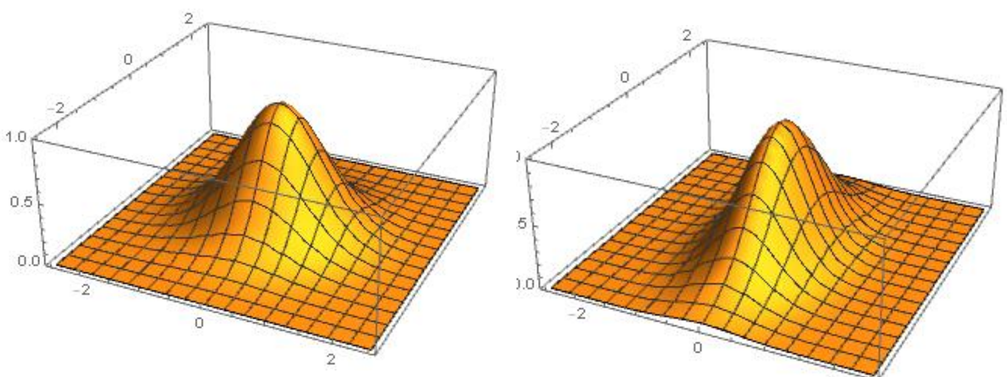
$$X_2 = \frac{i}{\sqrt{2}}\left(a^{\dagger} - a\right)$$

$$X_{\theta} = \frac{1}{\sqrt{2}}\left(ae^{-i\theta} + a^{\dagger}e^{i\theta}\right)$$

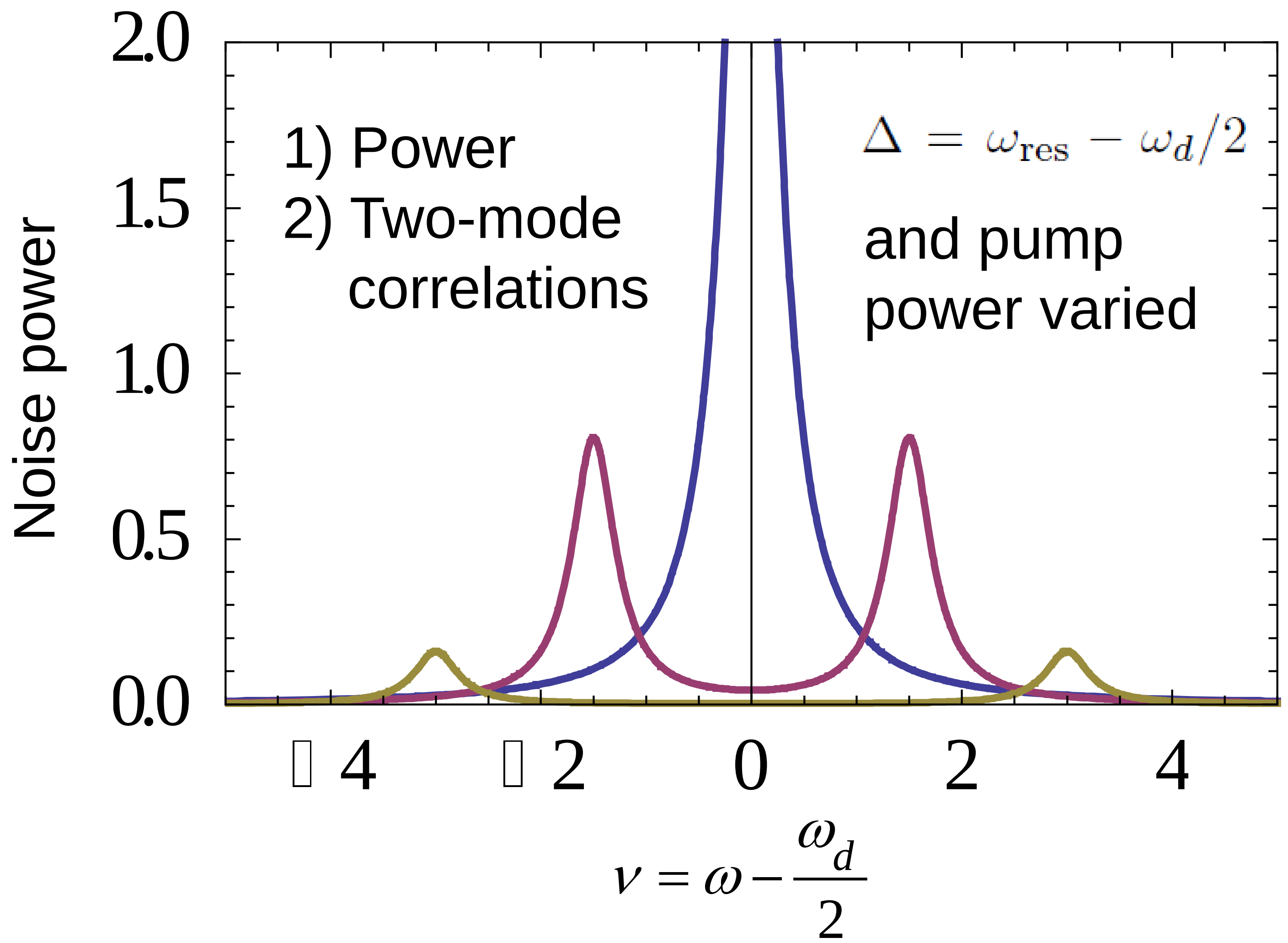
**Since** $\left[X_1, X_2\right] = i$**, there must be an uncertainty relation**

$$\Delta X_1 \Delta X_2 \geq \frac{1}{2}$$

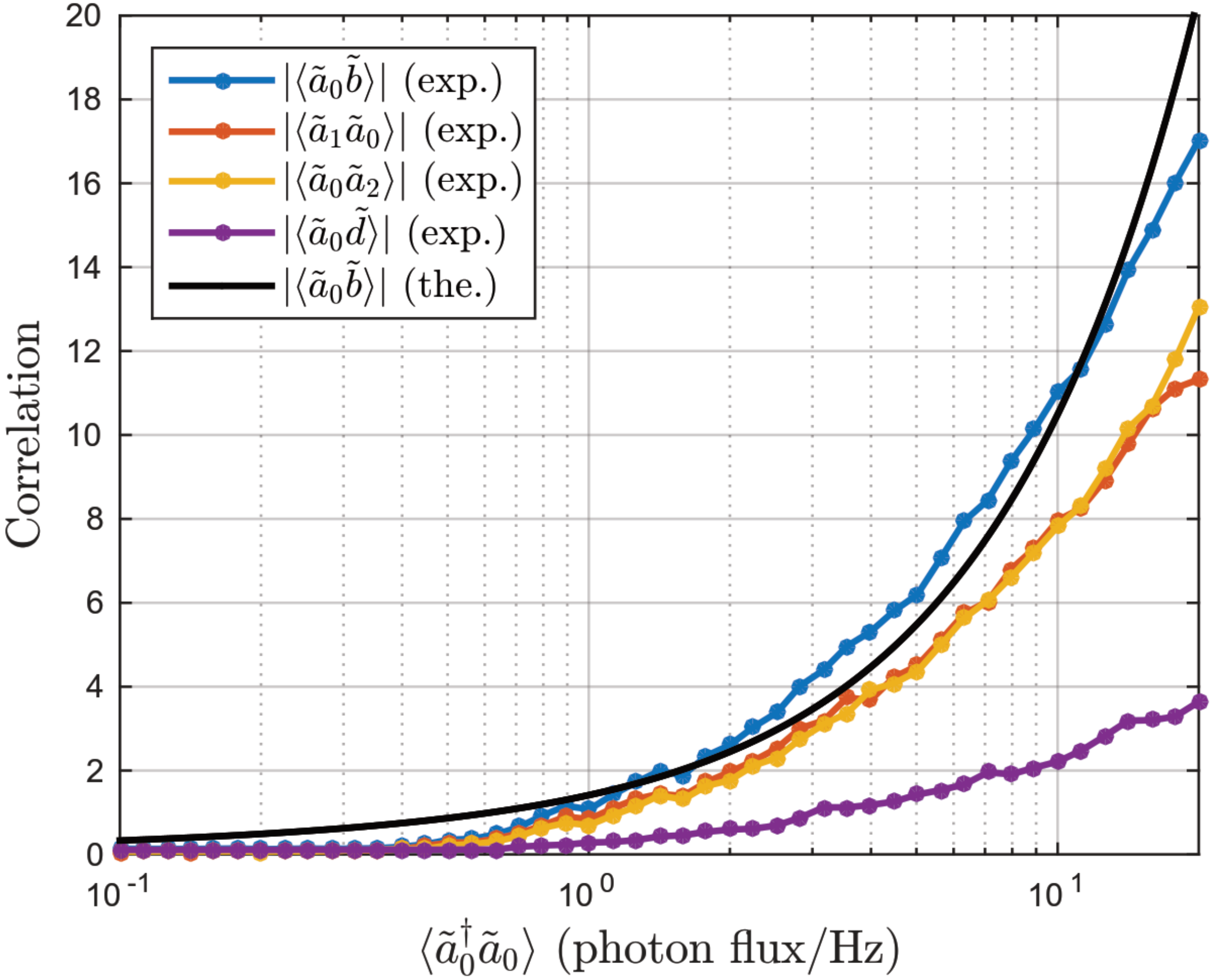
**Correlation of quadratures can be manipulated**

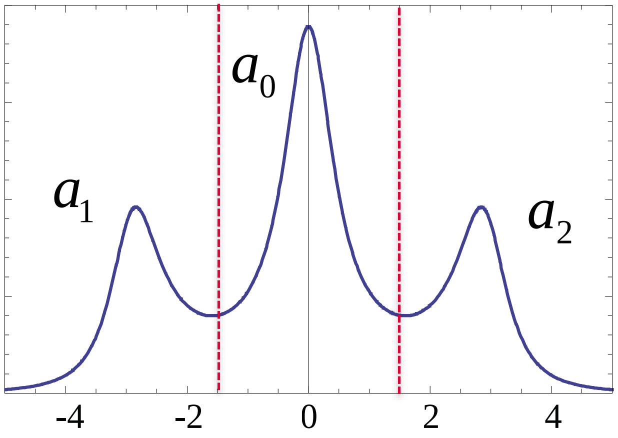


# Basic quantities



# Mode correlators I




$$\tilde{b} = \frac{1}{\sqrt{2}}\left(\tilde{a}_1 + \tilde{a}_2\right)$$

$$\tilde{d} = \frac{1}{\sqrt{2}}\left(\tilde{a}_1 - \tilde{a}_2\right)$$

# Field quantization

$$\nabla^2 \mathbf{A} = \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} \quad \mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}} \mathbf{A}_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t + i\mathbf{k}\cdot\mathbf{r}} + \mathbf{A}^*_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t - i\mathbf{k}\cdot\mathbf{r}}$$

## The energy stored in an EM field

$$H = \frac{1}{2}\int_V dV(\epsilon_0 \mathbf{E}^2 + \mu_0^{-1}\mathbf{B}^2)$$

## Energy for a single mode

$$H_{\mathbf{k}} = 2\epsilon_0 V \omega_{\mathbf{k}}^2 \mathbf{A}_{\mathbf{k}} \cdot \mathbf{A}^*_{\mathbf{k}}$$

## Rewriting A in terms of quadratures

$$\mathrm{A}_{\mathrm{k}} = \frac{1}{\sqrt{4\epsilon_0 V \omega_{\mathrm{k}}^2}}(\omega_{\mathrm{k}} X_{\mathrm{k}} + iP_{\mathrm{k}})\hat{\varepsilon}_{\mathrm{k}}$$

$$\Rightarrow \quad H_{\mathrm{k}} = \frac{1}{2}(P_{\mathrm{k}}^2 + \omega_{\mathrm{k}}^2 X_{\mathrm{k}}^2)$$

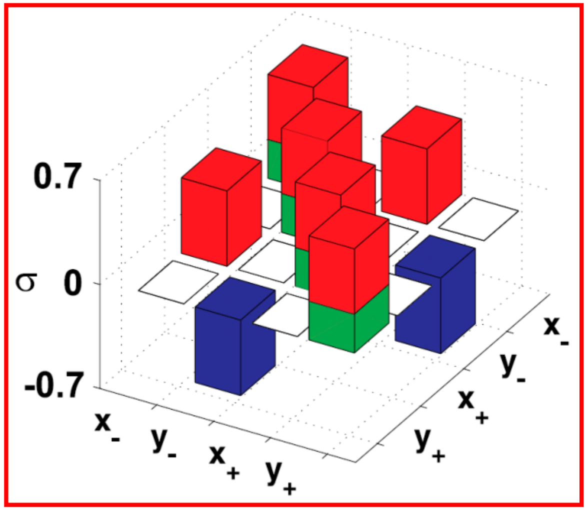
$$H_{\mathrm{k}} = \hbar\omega_{\mathrm{k}}(a_{\mathrm{k}}^\dagger a_{\mathrm{k}} + \frac{1}{2}))$$

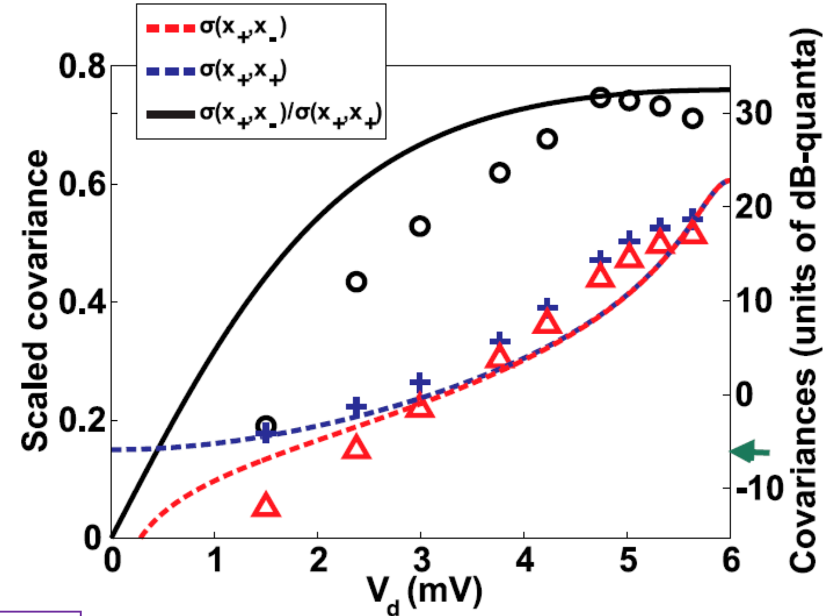
# Measurements of quadrature correlations

$$\sigma(A,B) = \langle AB + BA \rangle / 2$$

***Diagonal:***
*DCE*

***Off-diagonal:***
*Two-mode squeezing*




$$x_{\rm out}^{(\theta_+)}(\nu) = \frac{1}{2}\left(\tilde{a}_{\rm out}(\nu)e^{-i\theta_+/2} + \tilde{a}^\dagger_{\rm out}(-\nu)e^{+i\theta_+/2}\right)$$
$$y_{\rm out}^{(\theta_+)}(\nu) = \frac{1}{2i}\left(\tilde{a}_{\rm out}(\nu)e^{-i\theta_+/2} - \tilde{a}^\dagger_{\rm out}(-\nu)e^{+i\theta_+/2}\right)$$

$$\sigma(x_+, x_-) \propto \langle a_+ a_- \rangle$$

**Nonseparability (entangled state):**

$$\sigma(x_+, x_+) - \sigma(x_+, x_-) \leq 1/4$$

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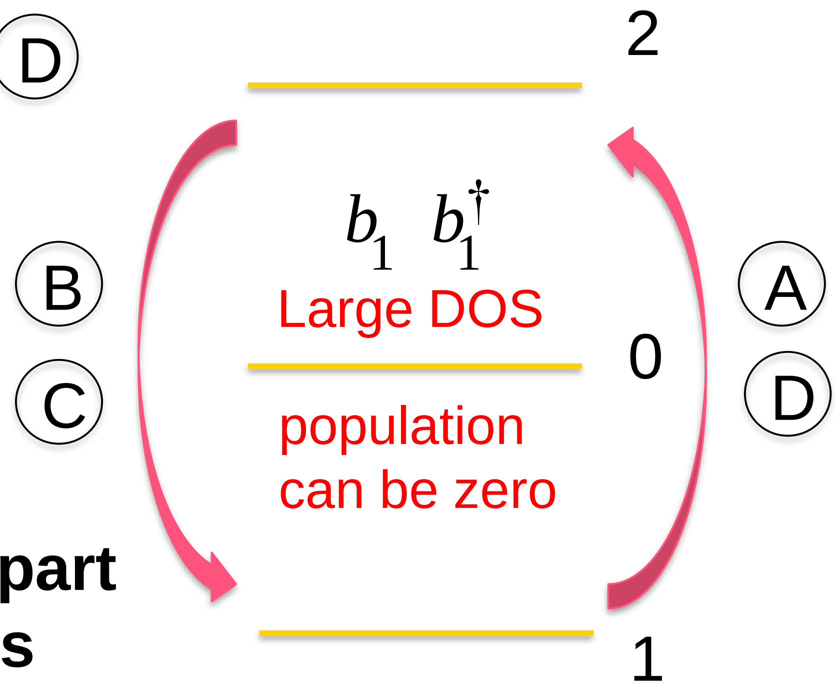
Lähteenmäki, P., et al.
Proc. Natl. Acad. Sci. **110**, 4234–4238 (2013).

# Vacuum induced coherence

Pump 1 $\quad H_1 = \underbrace{b_1 a_2^\dagger a_0^\dagger}_{A} + \underbrace{b_1^\dagger a_2 a_0}_{B}$

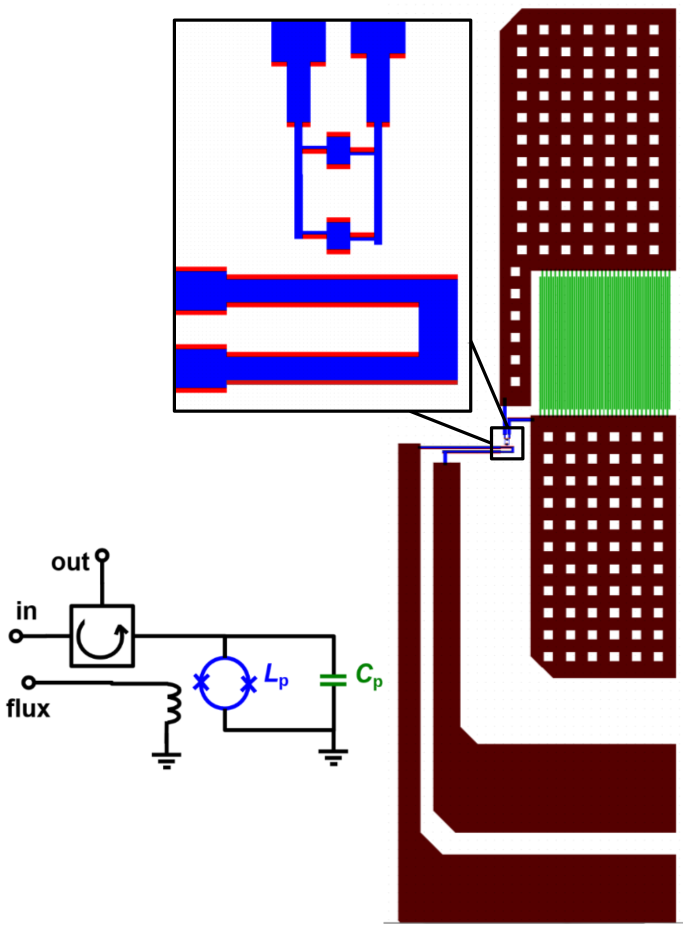
Pump 2 $\quad H_2 = \underbrace{b_2 a_0^\dagger a_1^\dagger}_{C} + \underbrace{b_2^\dagger a_0 a_1}_{D}$




Correct phase:
no time development
i.e. dark state

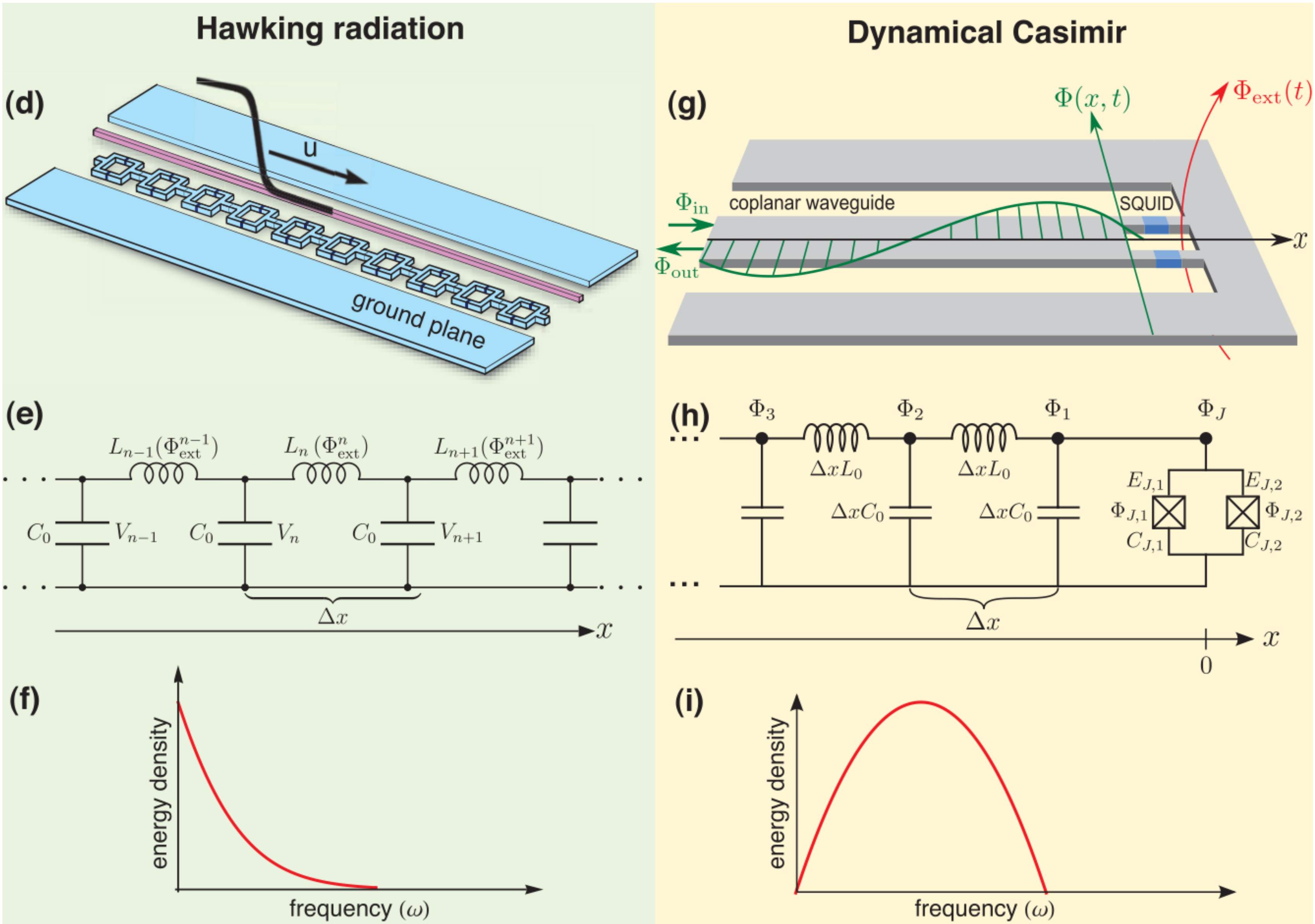
**Coherence due to the same quantum fluctuation taking part in the generation of the pairs**

# JPA Design




**Lumped element design**

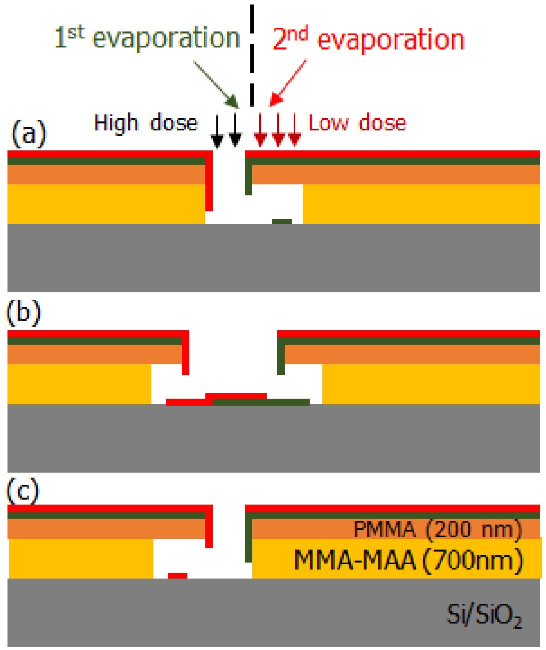
- Interdigital capacitor (drawn green)
  - Placed between bonding pads
  - Capacitance 1.2 pF
  - Area 300 × 330 μm²
- SQUID with 1.2 μA critical current
  - Josephson inductance ($\Phi = 0$): 275 pH
- Fluxline for DC and RF
  - Pump at double the signal frequency
- Low resonator impedance requires high critical current
  - Al/AlOx/Al junctions preferred
- Large junction area ( ~9 μm² )



Hawking radiation
Dynamical Casimir
(d)
u
ground plane
(e)
$L_{n-1}(\Phi^{n-1}_{\rm ext})$
$L_n(\Phi^n_{\rm ext})$
$L_{n+1}(\Phi^{n+1}_{\rm ext})$
$C_0$
$V_{n-1}$
$V_n$
$V_{n+1}$
$\Delta x$
$x$
(f)
energy density
frequency ($\omega$)
(g)
$\Phi(x,t)$
$\Phi_{\rm ext}(t)$
$\Phi_{\rm in}$
coplanar waveguide
SQUID
$\Phi_{\rm out}$
$x$
(h)
$\Phi_3$
$\Phi_2$
$\Phi_1$
$\Phi_J$
$\Delta x L_0$
$\Delta x C_0$
$E_{J,1}$
$E_{J,2}$
$\Phi_{J,1}$
$\Phi_{J,2}$
$C_{J,1}$
$C_{J,2}$
$\Delta x$
$x$
0
(i)
energy density
frequency ($\omega$)

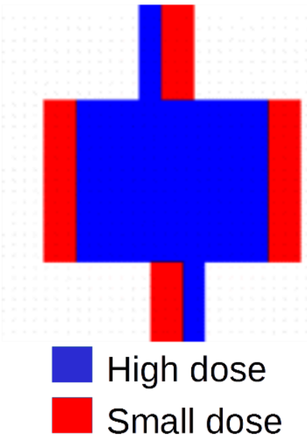
# JPA Fabrication

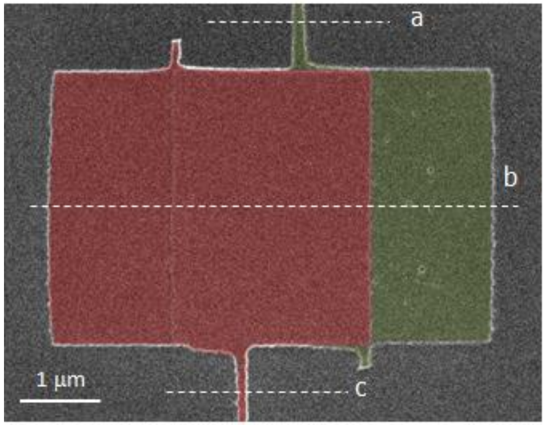
Common technique with a suspended bridge **limits** junction size




Junctions utilizing aluminum shadow evaporation **without** a suspended bridge

[F. Lecocq *et al*. Nanotechnology, 22, 315302 (2011).]




Double layer resist and using 100 kV e-beam lithography (reduce parasitic undercuts) with high and low doses

**The device is fabricated in one lithography step**

JJs, bonding pads, capacitors, fluxlines etc.

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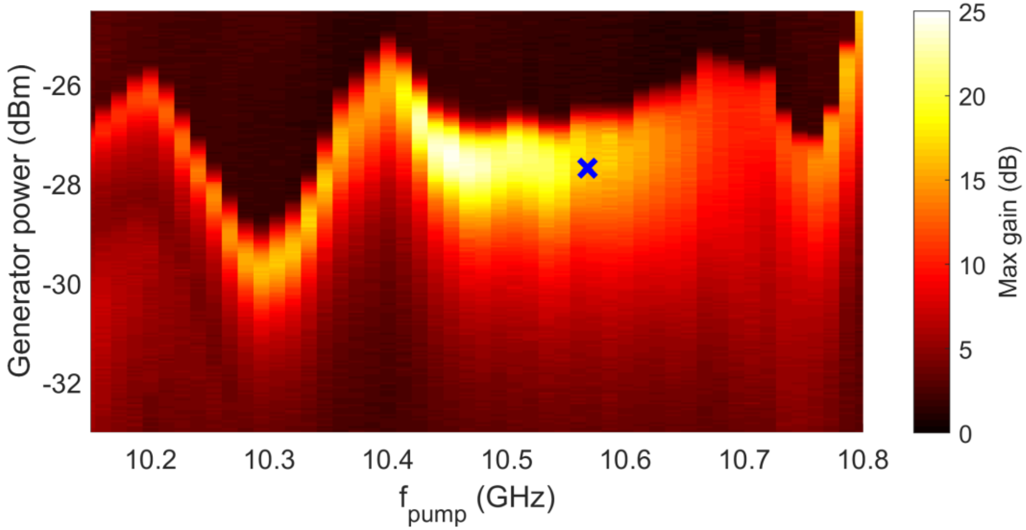
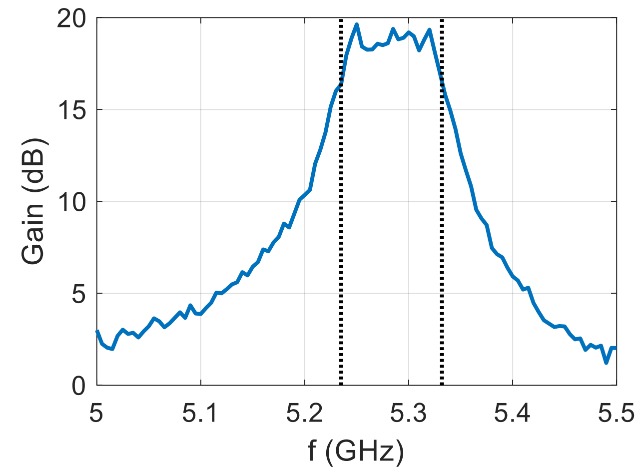
# Results – JPA Performance




- ❑ Maximum gain vs. pump frequency and power
  - ○ $f_{\text{pump}} = 2 \times f_{\text{signal}}$
- ❑ Tunable gain at single DC flux point
  - ○ I = 0.8 mA
- ❑ Additional tunability from DC flux
  - ○ Center frequency: 5 – 5.5 GHz

- ❑ Operating point example:
  - ○ ~20 dB gain
  - ○ 100 MHz bandwidth
    - • vertical lines
  - ○ 1 dB compression at -125 dBm

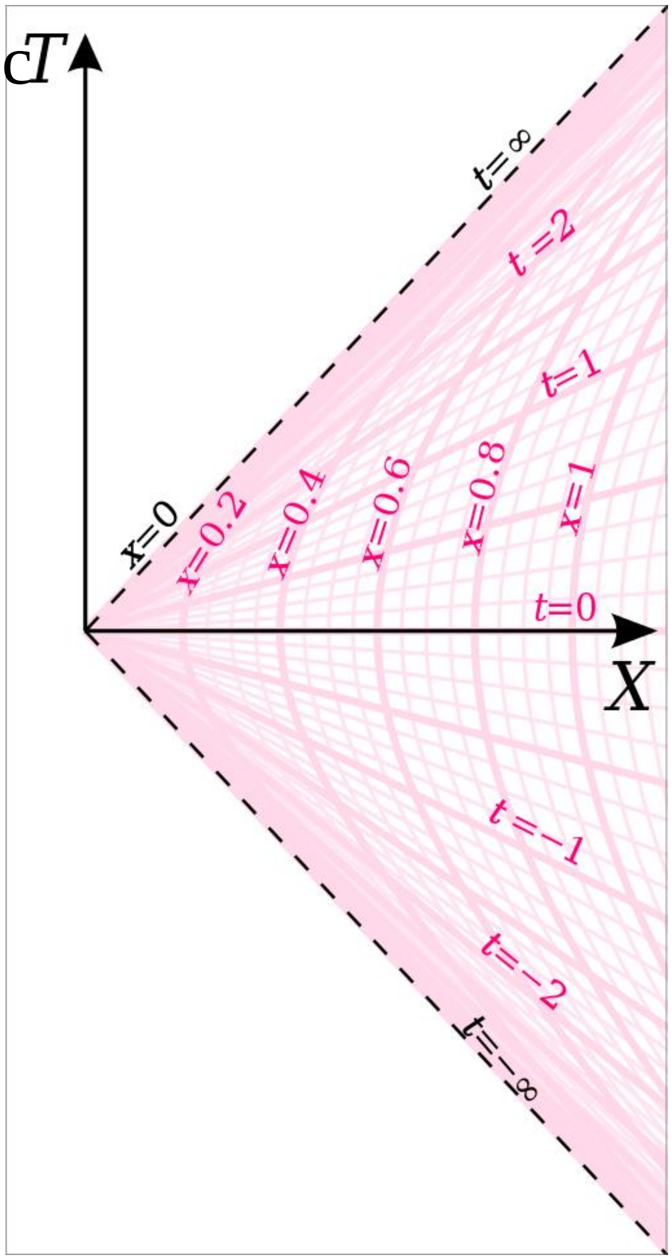
# Rindler coordinates

$$t = \frac{1}{\alpha}\operatorname{artanh}\left(\frac{T}{X}\right), \quad x = \sqrt{X^2 - T^2}, \quad y = Y, \quad z = Z$$

$$T = x\sinh(\alpha t), \quad X = x\cosh(\alpha t), \quad Y = y, \quad Z = z$$

$$t = \frac{c}{\alpha}\operatorname{artanh}\left(\frac{cT}{X}\right), \quad x = \sqrt{X^2 - (cT)^2}$$

$$T = \frac{x}{c}\sinh\left(\frac{\alpha t}{c}\right), \quad X = x\cosh\left(\frac{\alpha t}{c}\right)$$



# Acknowledgements



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Mikhail Perelshtein

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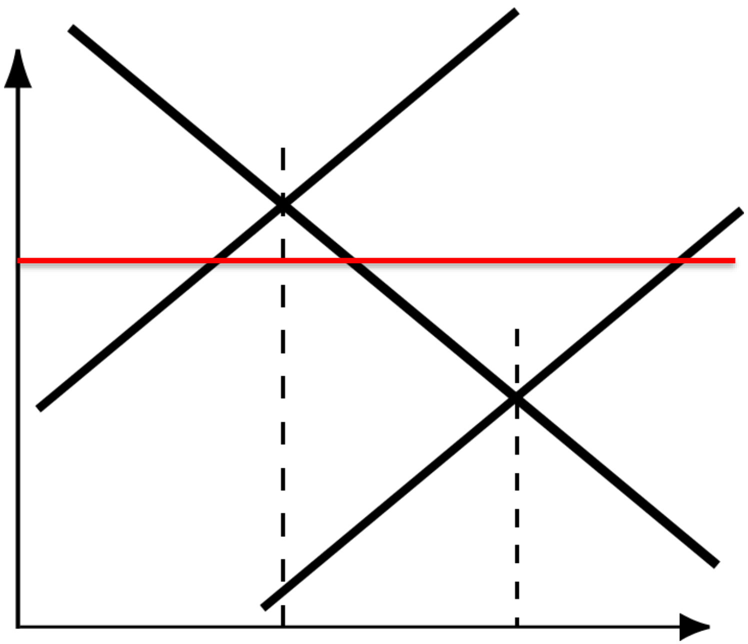
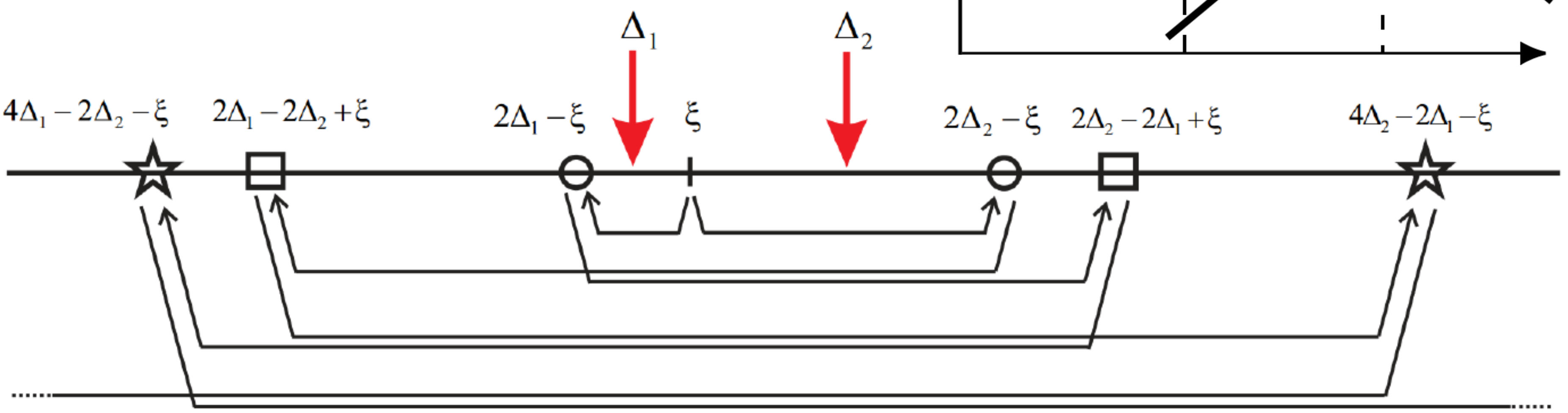
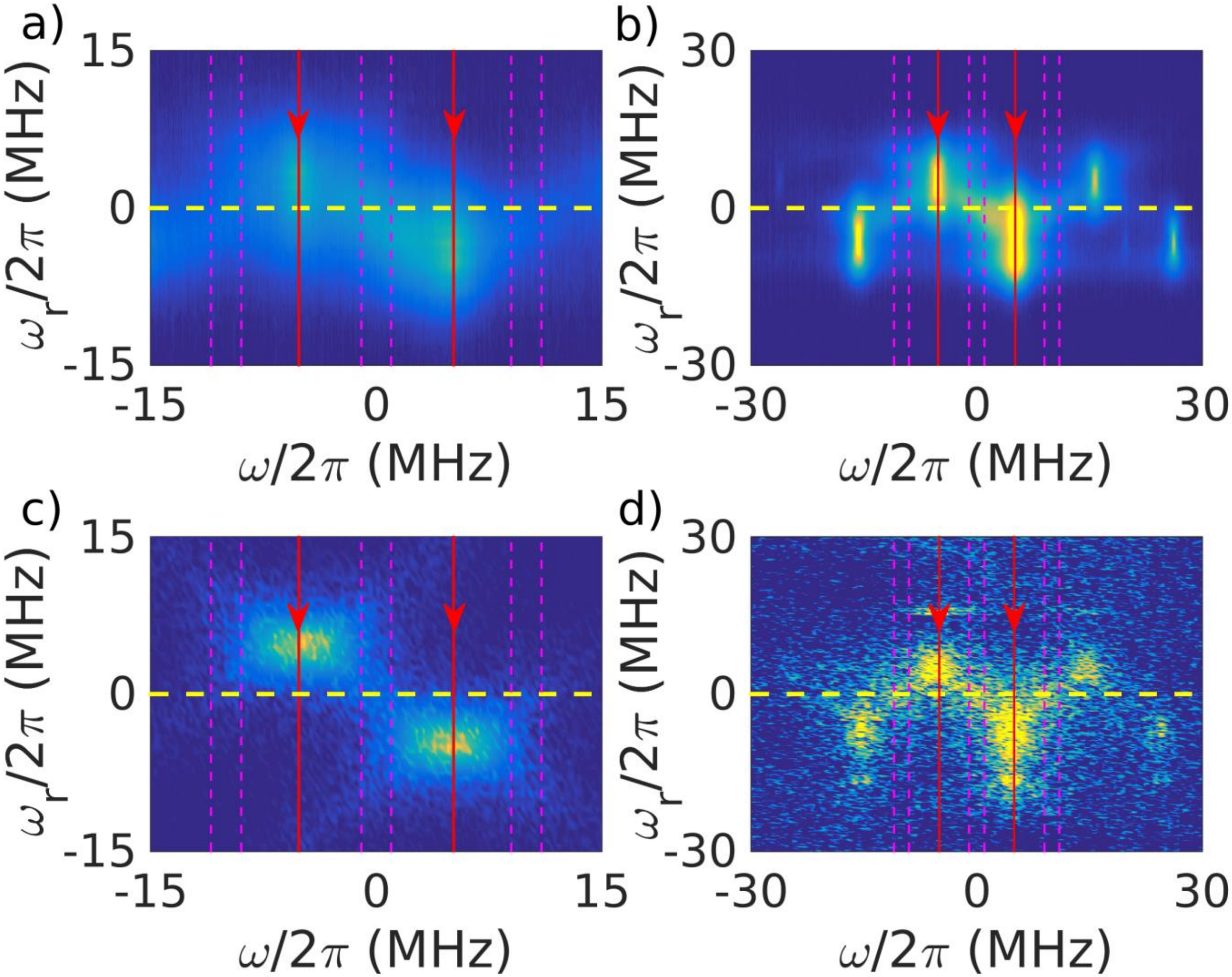
# Higher order correlations

- Reflections across the pump frequencies
- Importance goes down as distance to resonance frequencies increases





# Noise power measurements (low & high power)

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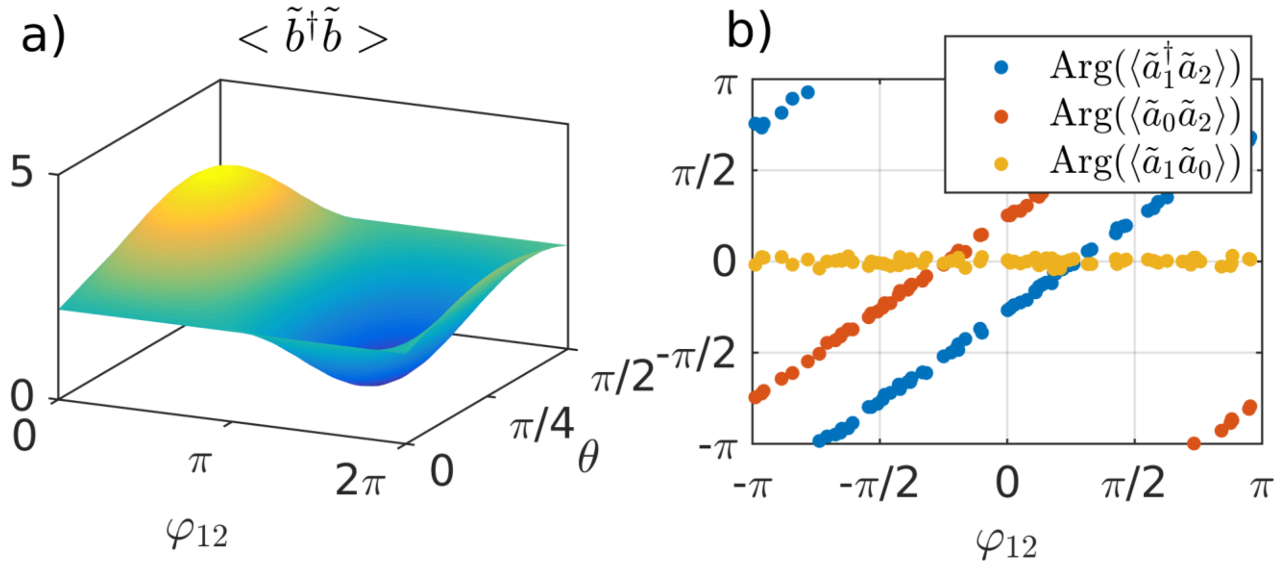
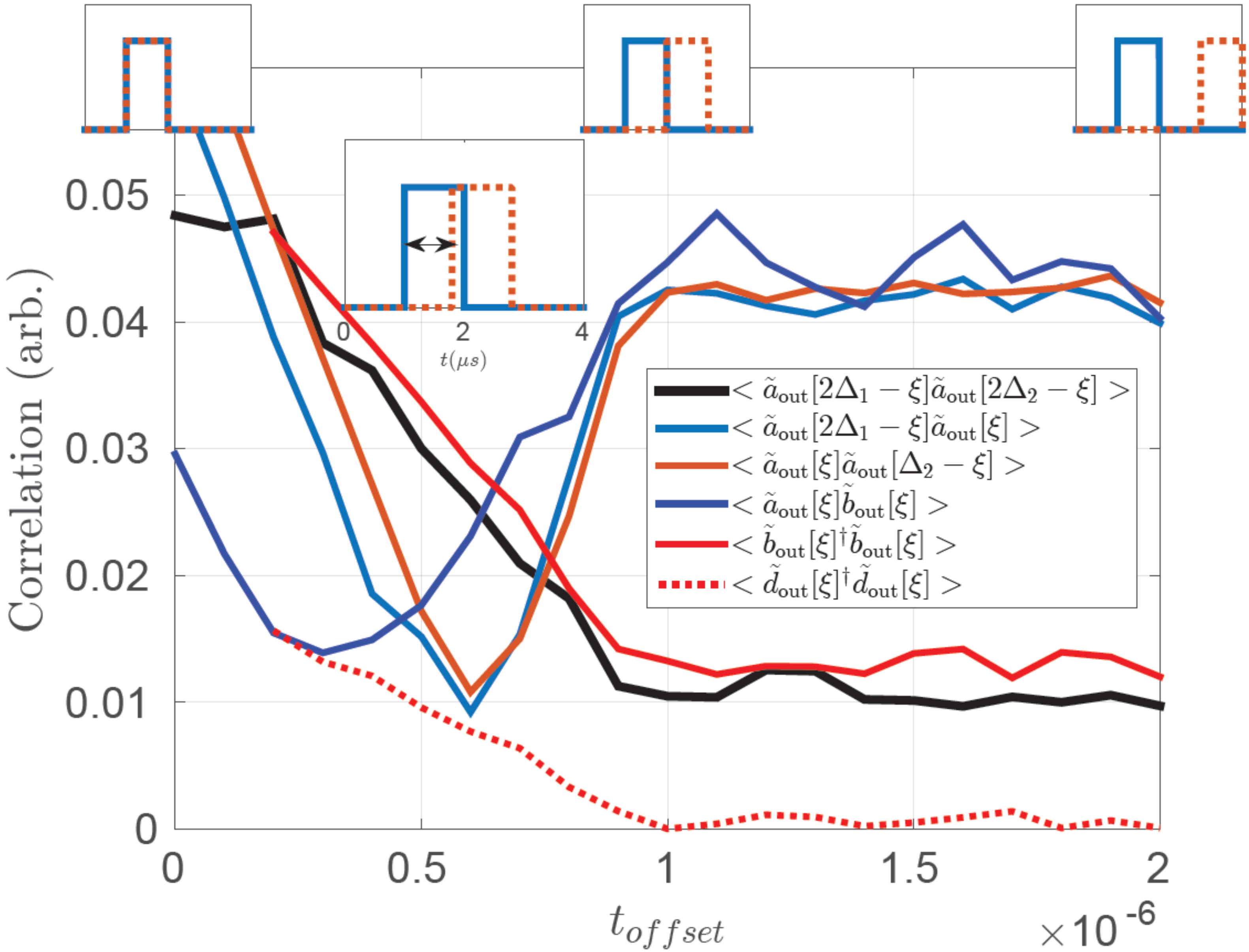
# Phase of the dark and bright states

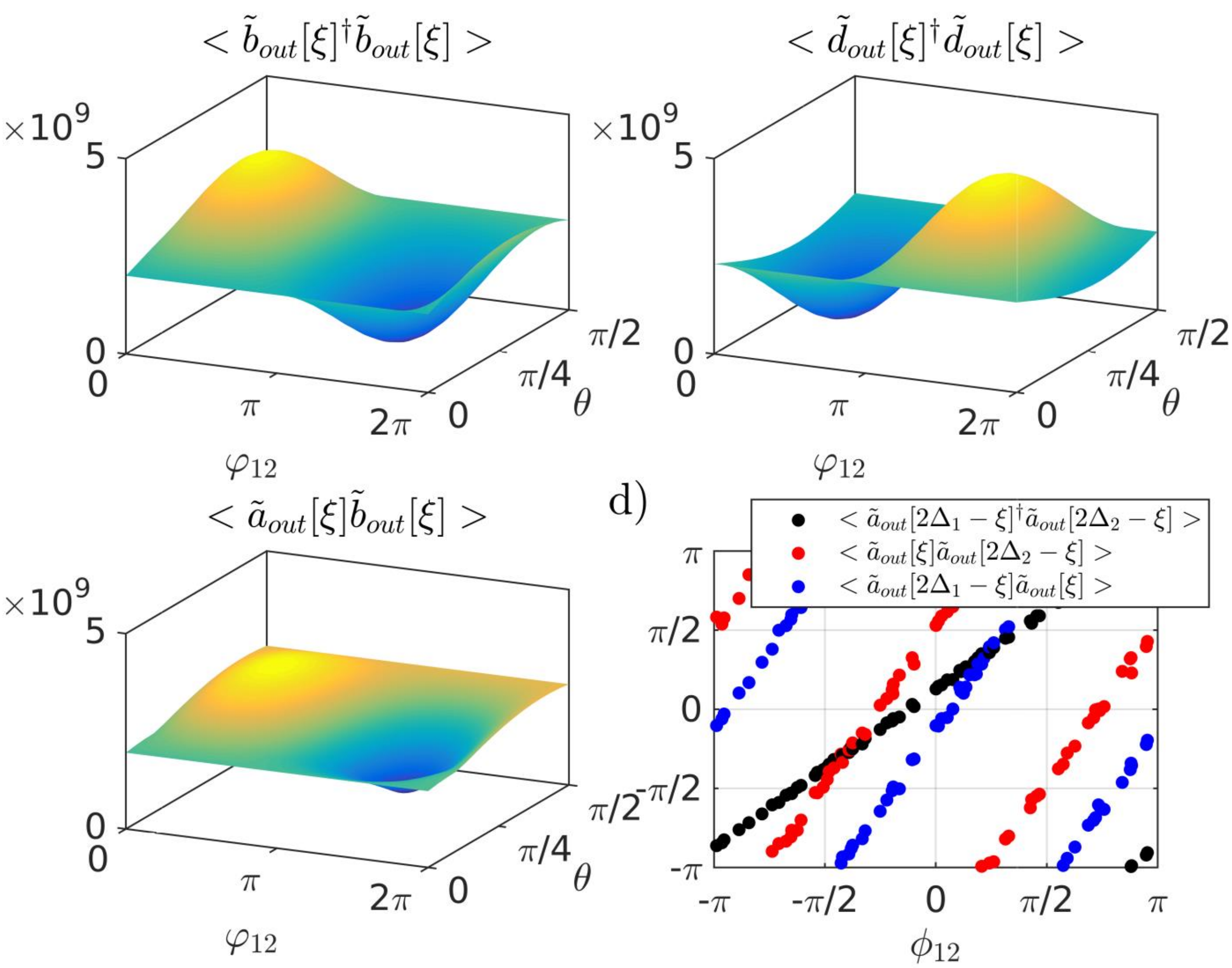



$$\tilde{b}[\xi] = \left\{ e^{-i\varphi_1} \cos\theta \, \tilde{a}[2\Delta_1 - \xi] + e^{-i\varphi_2} \sin\theta \, \tilde{a}[2\Delta_2 - \xi] \right\} \big/ \sqrt{2}$$

# Pulsed pumps with tuned overlap



# Phase of the dark and bright states




$$\tilde{b}[\xi] = \left\{ e^{-i\varphi_1} \cos\theta \, \tilde{a}[2\Delta_1 - \xi] + e^{-i\varphi_2} \sin\theta \, \tilde{a}[2\Delta_2 - \xi] \right\} / \sqrt{2}$$

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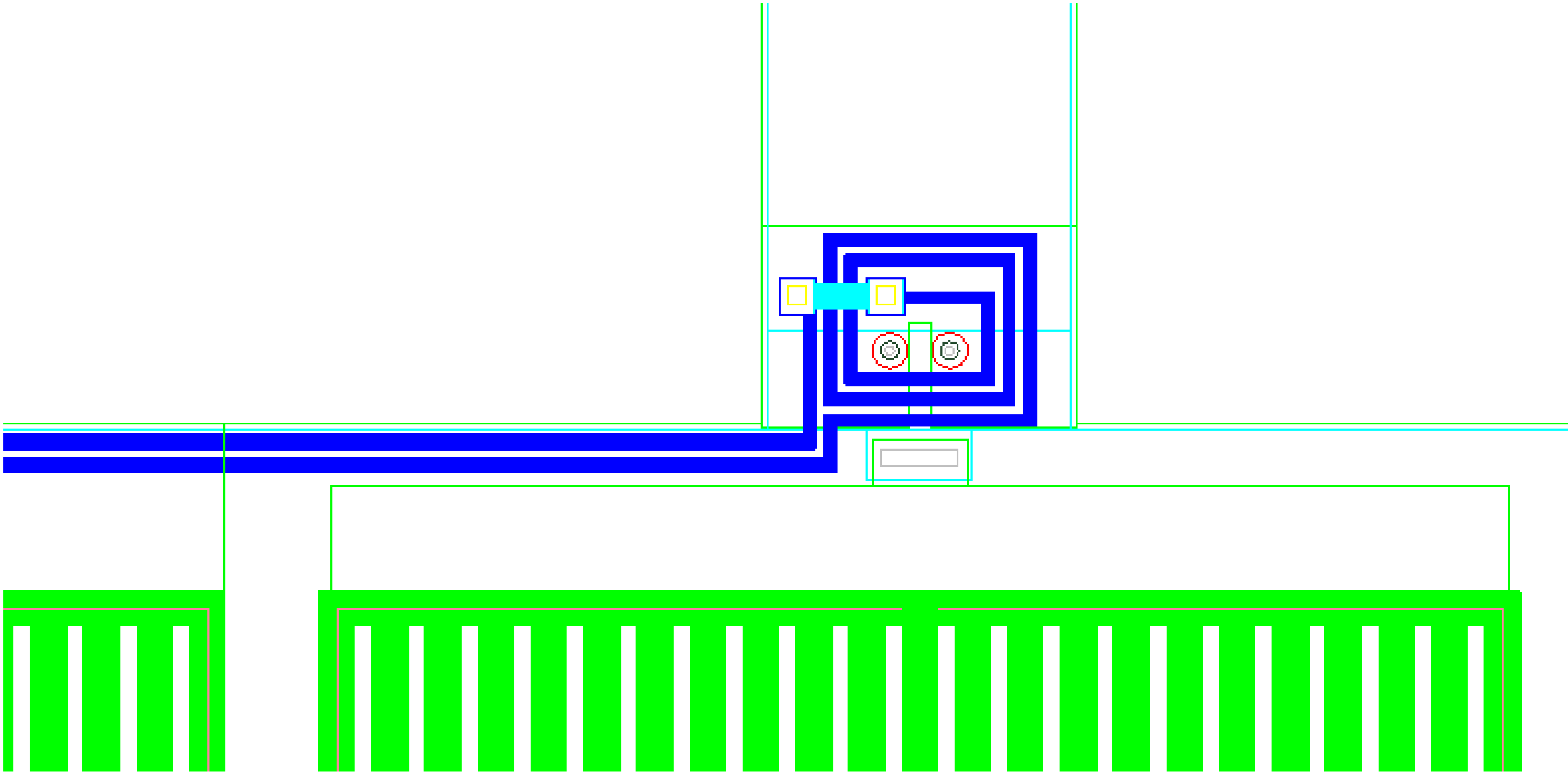
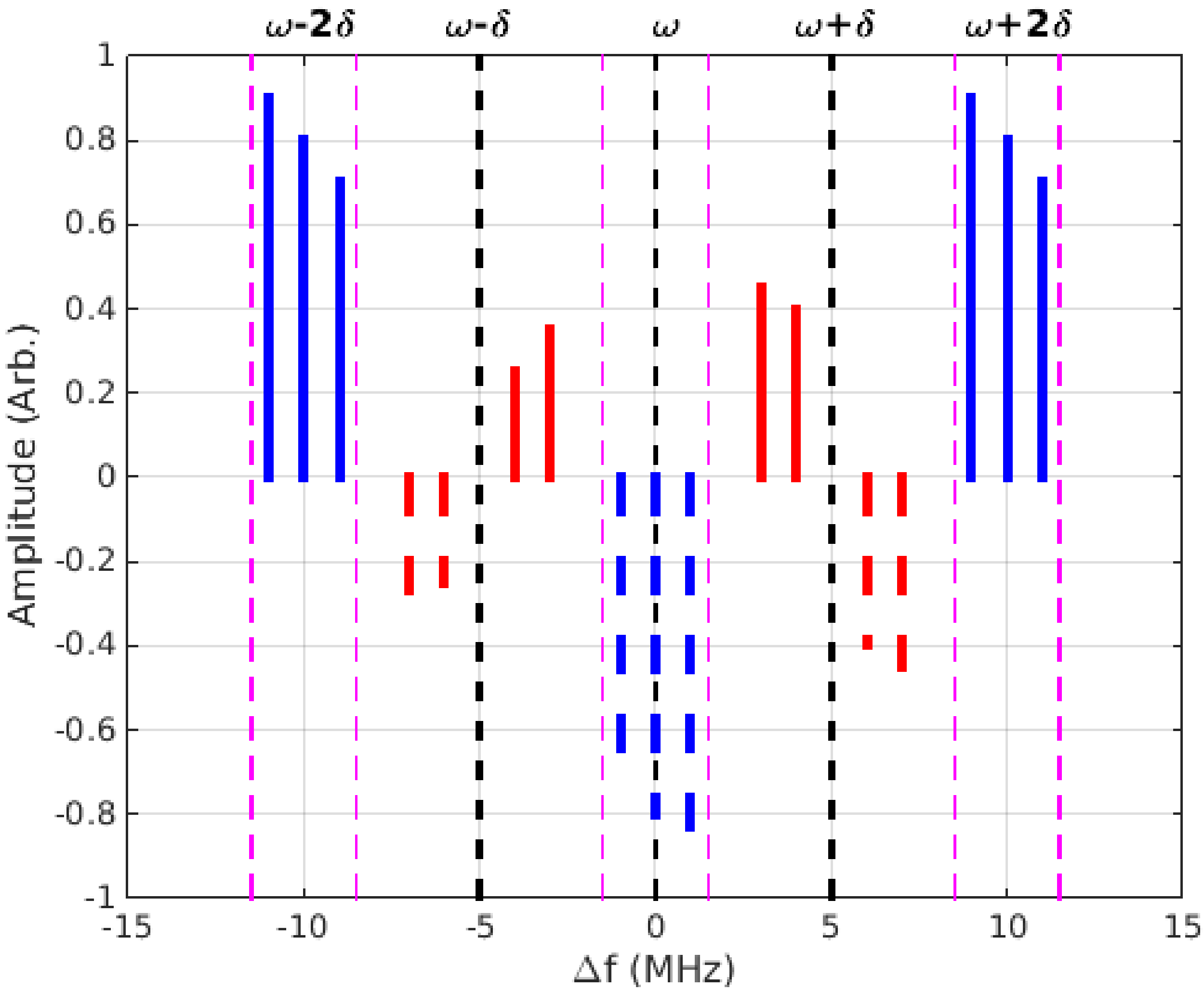
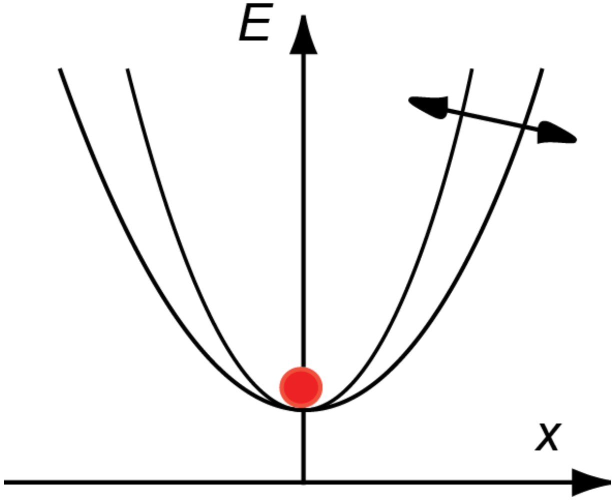
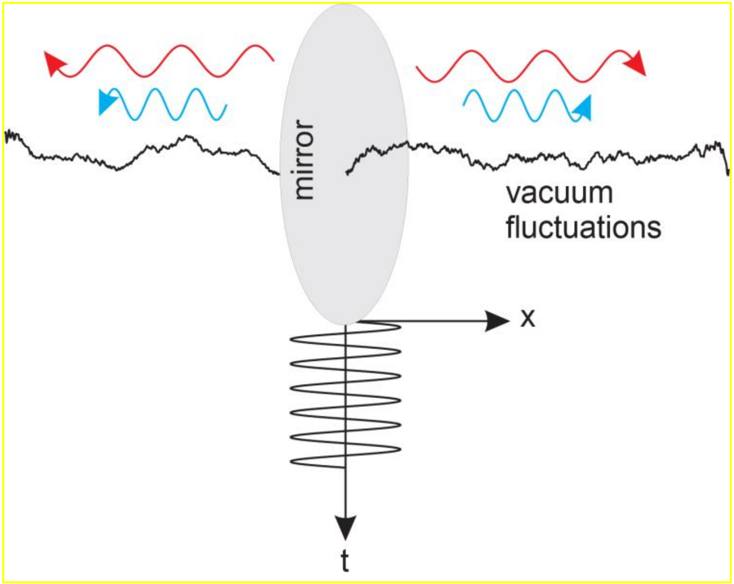



ω-2δ
ω-δ
ω
ω+δ
ω+2δ
Amplitude (Arb.)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-15
-10
-5
0
5
10
15
Δf (MHz)

# Classical versus quantum parametric excitation

$$\ddot{x} + \Omega_0^2 \left[1 + g\cos(\Omega_1 t)\right] x + \Gamma_0 \dot{x} = 0$$




- Classical vacuum cannot be parametrically excited.

- Quantum vacuum has inherent zero-point fluctuations, and can be parametrically excited.

# Correlators from Input/Output theory

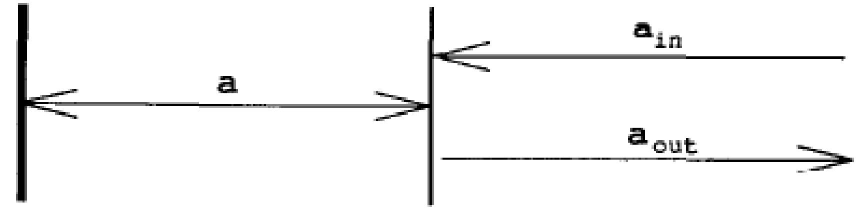
$$\tilde{a}_{\text{out}}(\nu) = \left[1 - \frac{\kappa\chi\left(\frac{\omega_d}{2}+\nu\right)}{\mathcal{N}(\nu)}\right]\tilde{a}_{\text{in}}(\nu) - \frac{i\alpha\kappa}{\mathcal{N}(\nu)}\chi\left(\frac{\omega_d}{2}+\nu\right)\chi\left(\frac{\omega_d}{2}-\nu\right)^*\tilde{a}^\dagger_{\text{in}}(\nu)$$

$$\langle \tilde{a}^\dagger_{\text{out}}(-\nu)\,\tilde{a}_{\text{out}}(\nu')\rangle = \text{THERM}(\nu)\delta(\nu-\nu') + \text{DCE}(\nu)\delta(\nu-\nu').$$

$$\langle \tilde{a}_{\text{out}}\tilde{a}_{\text{out}}\rangle_{T=0}(\nu) = \frac{i\alpha\kappa\chi\left(\frac{\omega_d}{2}+\nu\right)\chi^*\left(\frac{\omega_d}{2}-\nu\right)}{\mathcal{N}(\nu)}\left[-1 + \frac{\kappa}{\mathcal{N}(-\nu)^*}\chi\left(\frac{\omega_d}{2}-\nu\right)\right]$$

$$\nu = \omega - \omega_d/2 \qquad \Delta = \omega_{\text{res}} - \omega_d/2$$

# Input/Output theory




$$\tilde{H}_{\mathrm{RWA}} = \hbar\Delta a^\dagger a - \frac{\hbar}{2}(\alpha^* a^2 + \alpha a^{\dagger 2})$$

$$a(t) = \tilde{a}(t)\exp[-i\omega_d t/2]$$

$$\Delta = \omega_{\mathrm{res}} - \omega_d/2$$

$$\alpha = \frac{1}{4}\omega_n(\Phi_{\mathrm{bias}})e^{i\gamma_d}\tan\left(\frac{\pi\Phi_{\mathrm{bias}}}{\Phi_0}\right)\frac{\pi\delta\Phi_{\mathrm{ext}}}{\Phi_0}$$

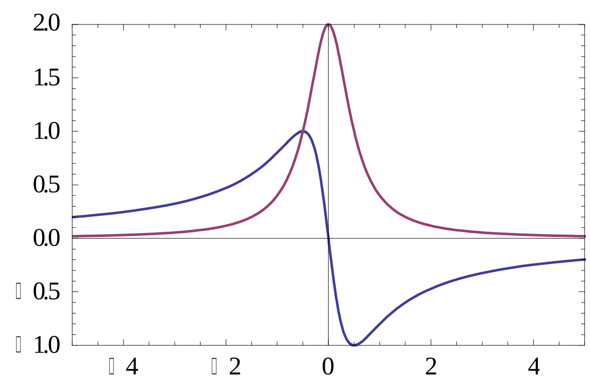
$$\dot{\tilde{a}} = -i\Delta\tilde{a} + i\alpha\tilde{a}^\dagger - \frac{\kappa}{2}\tilde{a} - \sqrt{\kappa}\tilde{a}_{\mathrm{in}}$$

$$\tilde{a}_{\mathrm{out}}(\nu) = \left[1 - \frac{\kappa\chi\left(\frac{\omega_d}{2}+\nu\right)}{\mathcal{N}(\nu)}\right]\tilde{a}_{\mathrm{in}}(\nu) - \frac{i\alpha\kappa}{\mathcal{N}(\nu)}\chi\left(\frac{\omega_d}{2}+\nu\right)\chi\left(\frac{\omega_d}{2}-\nu\right)^*\tilde{a}^\dagger_{\mathrm{in}}(\nu)$$

$$\nu = \omega - \omega_d/2$$

$$\chi(\omega) = [\kappa/2 - i(\omega - \omega_{\mathrm{res}})]^{-1}$$

$$\mathcal{N}(\nu) = 1 - |\alpha|^2\chi\left(\frac{\omega_d}{2}+\nu\right)\chi\left(\frac{\omega_d}{2}-\nu\right)^*$$



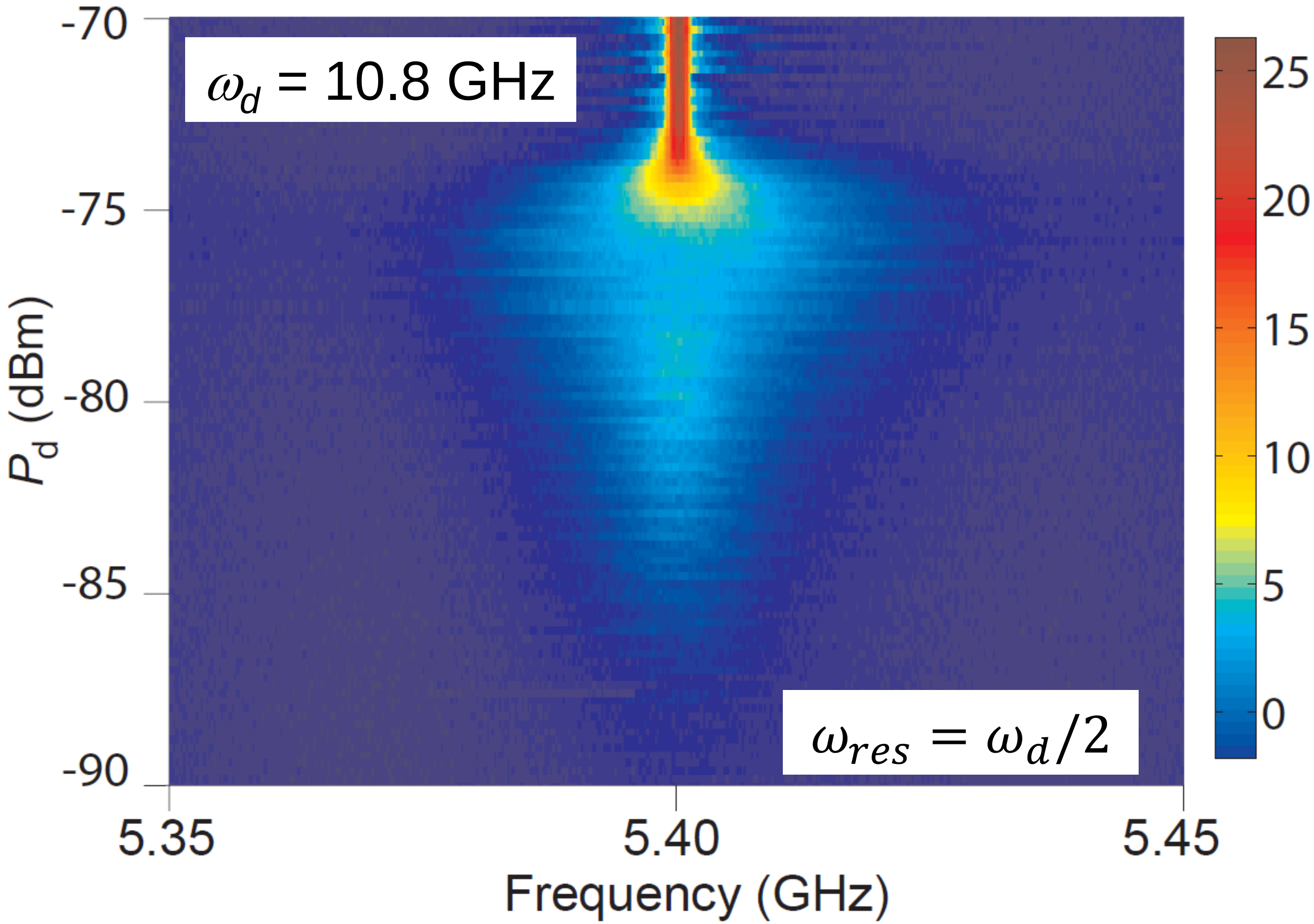
# Noise spectra with increased pump drive

Parametric instability at -75 dBm

$\delta L < 10$ mm

$$\frac{v}{c} < 0.5$$

0.1 photons/s per unit band

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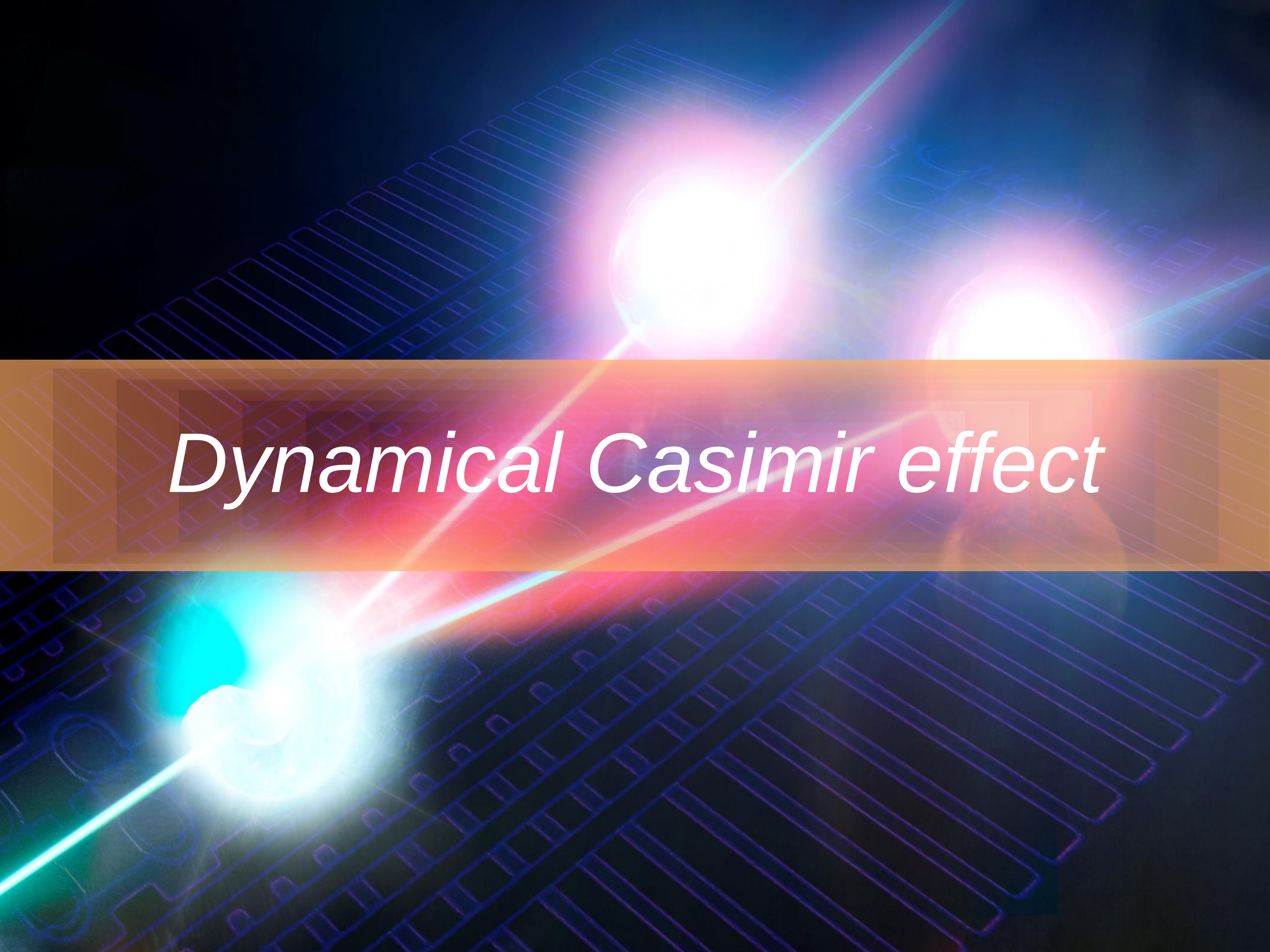

Dynamical Casimir effect

# Data analysis

$$\mathcal{F}[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \omega t} dt,$$

$$N = 2^{23} \approx 8\text{M}$$

$$F[\omega] = \sum_{t=0}^{N-1} f[t] \exp\left(\frac{-2\pi i \omega t}{N}\right),$$

$$(f^* \star \bar{g})[n]$$

$$(f \star g)[n] = \sum_{m=1}^{k} f^*[m] g[m+n]$$

$$z_{cor}[t] = \sum_{\tau=-\infty}^{\infty} \overline{f[\tau]} g[t+\tau],$$

$$(f \star \bar{g})[n] = \sum_{m=1}^{k} f^*[m] g[n-m]$$

$$z_{cor}[t] = IDFT\left[\frac{1}{N} X^*[f] \cdot Y[f]\right],$$

$$\bar{g}[m] = g[k-m]$$

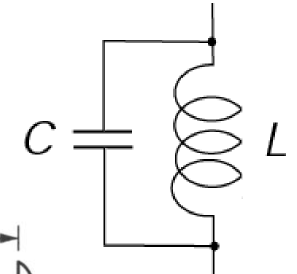
$$z_{cor}[t] = \frac{1}{M} IDFT\left[\sum_{k=1}^{M} \frac{1}{N} X_k^*[f] \cdot Y_k[f]\right],$$

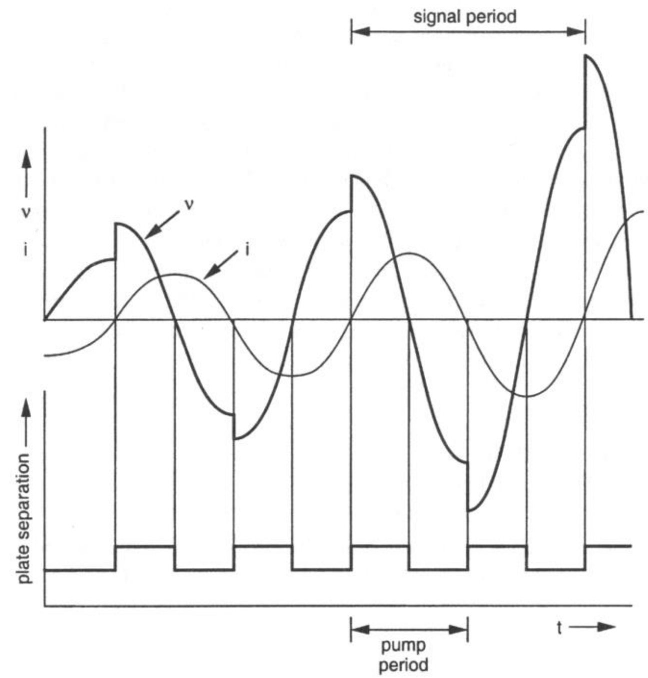
$$z_{cor}[\omega] = \frac{1}{M}\left[\sum_{k=1}^{M} \frac{1}{N} X_k^*[\omega] \cdot Y_k[\omega]\right],$$

# Parametric oscillation

Parametric oscillations can be:

- innocuous: e.g. child in a swing
- dangerous: e.g. bridges, container ships
- useful: e.g. low-noise parametric amplifiers



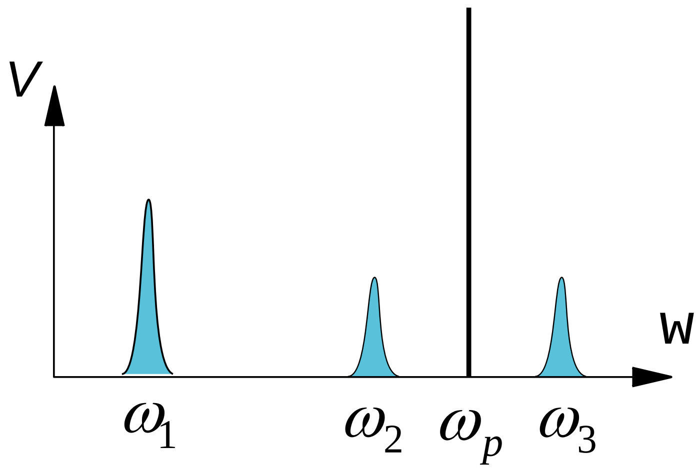

*L. Blackwell and K. Kotzebue, Semi-conductor-Diode Parametric Amplifiers*

The **Botafumeiro** is a famous thurible found in the **Santiago de Compostela Cathedral**..

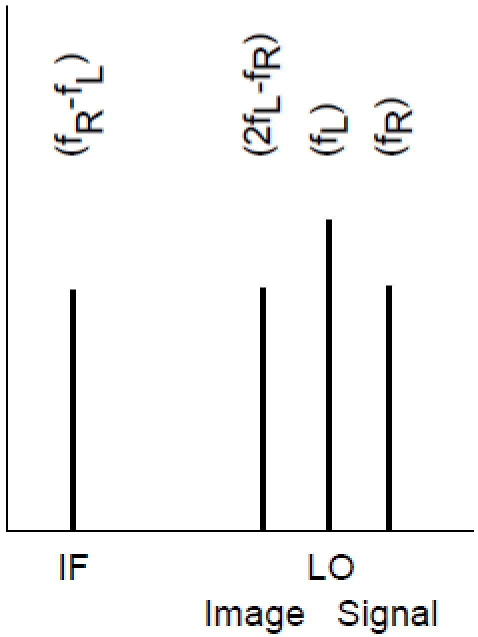
# Conversion matrix for parametric circuits

$$i = \frac{d}{dt}\left[C(t)v(t)\right]$$

$$v = V_1 \exp(-j\omega_1 t) + V_2 \exp(-j\omega_2 t) + V_3 \exp(-j\omega_3 t)$$




Mixer:

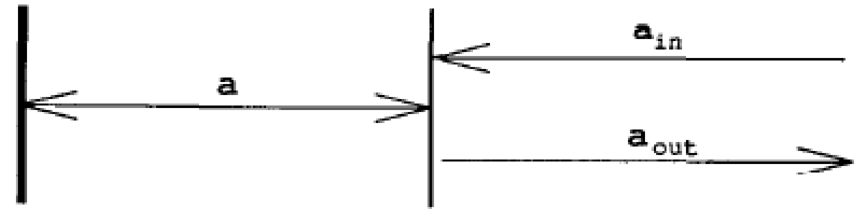



$$\begin{pmatrix} I_2^* \\ I_1 \\ I_3 \end{pmatrix} = \begin{pmatrix} -j\omega_2 C_0 & -j\omega_2 C_0 M & 0 \\ j\omega_1 C_0 M & j\omega_1 C_0 & j\omega_1 C_0 M \\ 0 & j\omega_2 C_0 M & j\omega_3 C_0 \end{pmatrix} \begin{pmatrix} V_2^* \\ V_1 \\ V_3 \end{pmatrix}$$

$$C(t) = C_0 (1 + M \cos \omega_p t)$$

# Input/Output theory

$$\tilde{H}_{\mathrm{RWA}} = \hbar\Delta a^\dagger a - \frac{\hbar}{2}(\alpha^* a^2 + \alpha a^{\dagger 2})$$




$$\dot{\tilde{a}} = -i\Delta\tilde{a} + i\alpha\tilde{a}^\dagger - \frac{\kappa}{2}\tilde{a} - \sqrt{\kappa}\tilde{a}_{\mathrm{in}}$$

$$a(t) = \tilde{a}(t)\exp[-i\omega_d t/2]$$
$$\Delta = \omega_{\mathrm{res}} - \omega_d/2$$

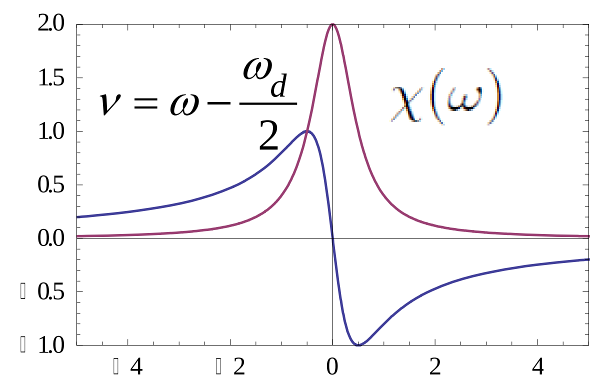
$$\tilde{a}^\dagger(\nu) = \int_{-\infty}^{\infty} dt\, \exp(i\nu t)\tilde{a}^\dagger(t) = \left[\tilde{a}(-\nu)\right]^\dagger$$

$$\tilde{a}_{\mathrm{out}}(\nu) = \left[1 - \frac{\kappa\chi\left(\frac{\omega_d}{2}+\nu\right)}{\mathcal{N}(\nu)}\right]\tilde{a}_{\mathrm{in}}(\nu) - \frac{i\alpha\kappa}{\mathcal{N}(\nu)}\chi\left(\frac{\omega_d}{2}+\nu\right)\chi\left(\frac{\omega_d}{2}-\nu\right)^*\tilde{a}^\dagger_{\mathrm{in}}(\nu)$$

$$\mathcal{N}(\nu) = 1 - |\alpha|^2\chi\left(\frac{\omega_d}{2}+\nu\right)\chi\left(\frac{\omega_d}{2}-\nu\right)^*$$

$$\tilde{a}_{out} = \cosh\lambda\,\tilde{a}_{in} - \sinh\lambda\,\tilde{a}^\dagger_{in} \quad \left(\alpha \propto \tanh\lambda/2\right)$$

*- Bogolyubov transformation*

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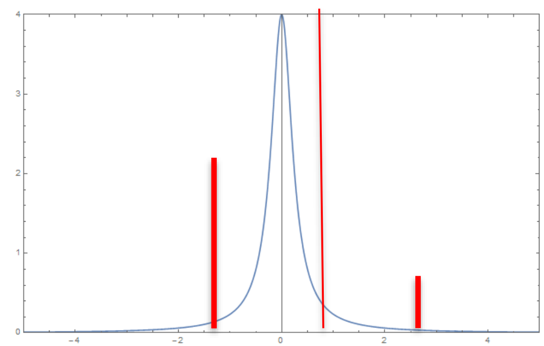
# Correlators from Input/Output theory

$$\tilde{a}_{\text{out}}(\nu) = \left[1 - \frac{\kappa\chi\left(\frac{\omega_d}{2}+\nu\right)}{\mathcal{N}(\nu)}\right]\tilde{a}_{\text{in}}(\nu) - \frac{i\alpha\kappa}{\mathcal{N}(\nu)}\chi\left(\frac{\omega_d}{2}+\nu\right)\chi\left(\frac{\omega_d}{2}-\nu\right)^* \tilde{a}^\dagger_{\text{in}}(\nu)$$

*Dynamical Casimir power:*

$$\left\langle \tilde{a}^\dagger_{out}(-\nu)\tilde{a}_{out}(\nu')\right\rangle = \mathrm{DCE}(\nu)\delta(\nu-\nu')$$




$$\nu = \omega - \omega_d/2$$

*Squeezing correlations:*

$$\langle \tilde{a}_{\text{out}}\tilde{a}_{\text{out}}\rangle_{T=0}(\nu) = \frac{i\alpha\kappa\chi\left(\frac{\omega_d}{2}+\nu\right)\chi^*\left(\frac{\omega_d}{2}-\nu\right)}{\mathcal{N}(\nu)}\left[-1+\frac{\kappa}{\mathcal{N}(-\nu)^*}\chi\left(\frac{\omega_d}{2}-\nu\right)\right]$$

# Field quantization

**From these derive wave equation for the vector potential**

$$\nabla^2 \mathbf{A} = \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2}$$

**Spatial mode expansion (exact form depends on boundary conditions)**

$$\mathbf{A}(\mathbf{r},t) = \sum_{\mathbf{k}} \mathbf{A}_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t + i\mathbf{k}\cdot\mathbf{r}} + \mathbf{A}^*_{\mathbf{k}} e^{i\omega_{\mathbf{k}} t - i\mathbf{k}\cdot\mathbf{r}}$$

$$\omega_{\mathbf{k}} = c|\mathbf{k}|$$

Plane wave solutions
Periodic BC, cubic volur

# Field quantization

## Promote the classical parameters to operators

$$
\begin{aligned}
\mathbf{A}_{\mathbf{k}} &= \frac{1}{\sqrt{4\epsilon_0 V \omega_k^2}} \left(\omega_k X_{\mathbf{k}} + i P_{\mathbf{k}}\right) \hat{\varepsilon}_{\mathbf{k}} \\
&\to \frac{1}{\sqrt{4\epsilon_0 V \omega_k^2}} \left(\omega_k x_{\mathbf{k}} + i p_{\mathbf{k}}\right) \hat{\varepsilon}_{\mathbf{k}} = \sqrt{\frac{\hbar}{2\epsilon_0 V \omega_{\mathbf{k}}}} a_{\mathbf{k}} \hat{\varepsilon}_{\mathbf{k}} \\
\mathbf{A}_{\mathbf{k}}{}^{*} &= \frac{1}{\sqrt{4\epsilon_0 V \omega_k^2}} \left(\omega_k X_{\mathbf{k}} - i P_{\mathbf{k}}\right) \hat{\varepsilon}_{\mathbf{k}} \\
&\to \frac{1}{\sqrt{4\epsilon_0 V \omega_k^2}} \left(\omega_k x_{\mathbf{k}} - i p_{\mathbf{k}}\right) \hat{\varepsilon}_{\mathbf{k}} = \sqrt{\frac{\hbar}{2\epsilon_0 V \omega_{\mathbf{k}}}} a_{\mathbf{k}}^{\dagger} \hat{\varepsilon}_{\mathbf{k}}.
\end{aligned}
$$

$$
\begin{aligned}
[x_{\mathbf{k}}, x_{\mathbf{k}'}] = 0, \quad [p_{\mathbf{k}}, p_{\mathbf{k}'}] &= 0, \quad [x_{\mathbf{k}}, p_{\mathbf{k}'}] = i\hbar\delta_{\mathbf{k}\mathbf{k}'} \\
[a_{\mathbf{k}}, a_{\mathbf{k}'}] = 0, \quad \left[a_{\mathbf{k}}^{\dagger}, a_{\mathbf{k}'}^{\dagger}\right] &= 0, \quad \left[a_{\mathbf{k}}, a_{\mathbf{k}'}^{\dagger}\right] = \delta_{\mathbf{k}\mathbf{k}'}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathbf{A}}_{\mathbf{k}} &= \sqrt{\frac{\hbar}{2\epsilon_0 V \omega_{\mathbf{k}}}} \hat{\varepsilon}_{\mathbf{k}} \left(a_{\mathbf{k}} e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{r}} + a_{\mathbf{k}}^{\dagger} e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{r}}\right) \\
\hat{\mathbf{E}}_{\mathbf{k}} &= i\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}} \hat{\varepsilon}_{\mathbf{k}} \left(a_{\mathbf{k}} e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{r}} - a_{\mathbf{k}}^{\dagger} e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{r}}\right) \\
\hat{\mathbf{B}}_{\mathbf{k}} &= i\sqrt{\frac{\hbar}{2\epsilon_0 V \omega_k}} \mathbf{k} \times \hat{\varepsilon}_{\mathbf{k}} \left(a_{\mathbf{k}} e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{r}} - a_{\mathbf{k}}^{\dagger} e^{i\omega_k t - i\mathbf{k}\cdot\mathbf{r}}\right)
\end{aligned}
$$

# Field quantization

**And find the energy for each mode**

$$H_{\mathbf{k}} = \frac{1}{2}\int_V dV(\epsilon_0 \hat{\mathbf{E}}_{\mathbf{k}}^2 + \mu_0^{-1}\hat{\mathbf{B}}_{\mathbf{k}}^2)$$

**Which simplifies to**

$$H_{\mathbf{k}} = \hbar\omega_{\mathbf{k}}(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2})$$

# Coherent states

**Defined as eigenstates of lowering operator**

$$a|\alpha\rangle = \alpha|\alpha\rangle \qquad |\alpha\rangle = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

**a is not Hermitian so *α* can be complex**

**Uncertainties in mode variables:**

$$\Delta q = \Delta p = \sqrt{\frac{1}{2}} \qquad \Delta q \Delta p = \frac{1}{2}$$

**Min uncertainty, equal between *q* and *p***

# Two-mode squeezed vacuum

The commutator

$$[q_2, p_2] = \frac{1}{2}[q_a + q_b, p_a + p_b] \\ = i$$

And so we have the same uncertainty relation between these joint observables as the quadratures themselves:

$$\Delta q_2 \Delta p_2 = \frac{1}{2}$$

# Two-mode squeezed vacuum

We can calculate the uncertainty in these observables for the TMSV

Recall

$$\Delta q_2 = \sqrt{\langle q_2^2\rangle - \langle q_2\rangle^2}$$

To calculate this requires several applications of the squeeze operator identities, ex.,

$$\begin{aligned} \langle a^\dagger b^\dagger\rangle &= \langle 0|S^\dagger a S S^\dagger b S|0\rangle \\ &= \langle 0|(a^\dagger \cosh r - e^{i\theta} b \sinh r)(b^\dagger \cosh r - e^{i\theta} a \sinh r)|0\rangle \end{aligned}$$

# Two-mode squeezed vacuum

$$\Delta q_2 = \frac{1}{\sqrt{2}}\sqrt{\sinh^2 r + \cosh^2 r - 2\cosh r \sinh r \cos\theta}$$

$$\Delta p_2 = \frac{1}{\sqrt{2}}\sqrt{\sinh^2 r + \cosh^2 r + 2\cosh r \sinh r \cos\theta}$$

**Choosing**

$$\theta = 0$$

$$\Delta q_2 = e^{-r}/\sqrt{2}$$

$$\Delta p_2 = e^{+r}/\sqrt{2}$$

**We can “squee one observable at the expense of the other**

# Two-mode squeezed vacuum

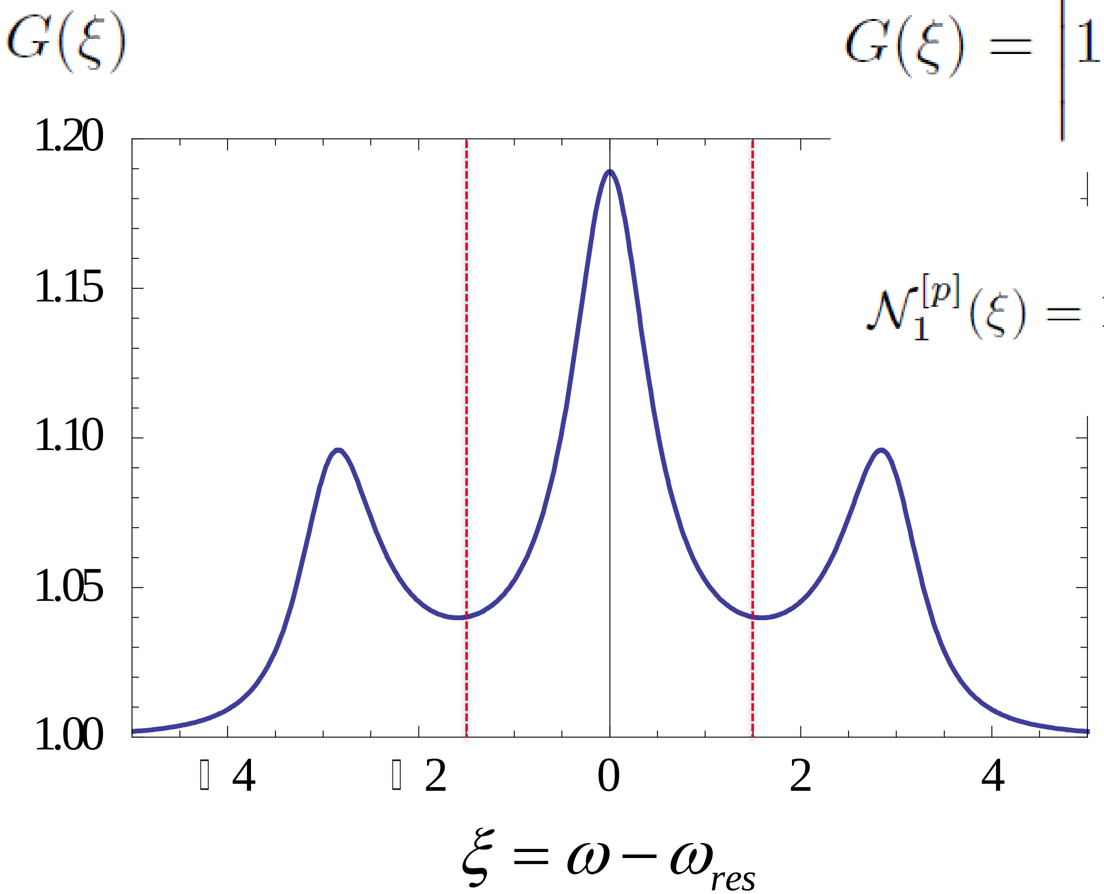
**The interesting properties show up in the correlations between quadrature obs.**

$$
\begin{aligned}
q_2 &= \frac{1}{\sqrt{2}}(q_a + q_b) \\
&= \frac{1}{2}(a + a^\dagger + b + b^\dagger) \\
p_2 &= \frac{1}{\sqrt{2}}(p_a + p_b) \\
&= \frac{i}{2}(a^\dagger - a + b^\dagger - b)
\end{aligned}
$$

$$
\Delta q_2 \Delta p_2 = \frac{1}{2}
$$

$$
\begin{aligned}
\Delta q_2 &= e^{-r}/\sqrt{2} \\
\Delta p_2 &= e^{+r}/\sqrt{2}
\end{aligned}
$$

# Parametric gain with two pumps

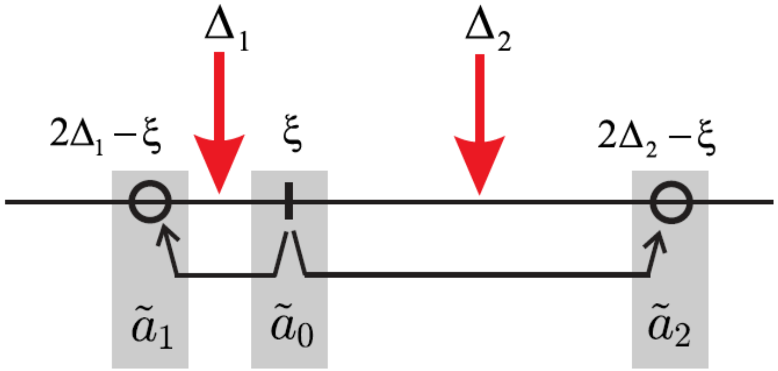



$$G(\xi) = \left| 1 + \frac{\kappa \chi(\xi)}{1 - \sum_{p=1}^{2} \mathcal{N}_1^{[p]}(\xi)} \right|^2$$

$$\mathcal{N}_1^{[p]}(\xi) = 1 - \frac{|\alpha_p|^2 \chi(\xi) \chi^*(2\Delta_p - \xi)}{\mathcal{N}_2^{[p]}(\xi)}$$

$$\xi = \omega - \omega_{res}$$

# Solution with two pumps




Iterative solution:

$$\tilde{a}_{\text{out}}[\xi] = \left[1 - \frac{\kappa\chi(\xi)}{1-\sum_{p=1}^{2}\mathcal{N}_1^{[p]}(\xi)}\right] a_{\text{in}}[\xi] -$$

$$-\frac{\kappa\chi(\xi)}{1-\sum_{p=1}^{2}\mathcal{N}_1^{[p]}(\xi)} \sum_{p=1}^{2} \frac{\alpha_p \chi^*(2\Delta_p - \xi)}{\mathcal{N}_2^{[p]}(\xi)} \left[(\tilde{a}_{\text{in}}[2\Delta_p - \xi])^\dagger + \frac{\alpha_{\bar{p}}^* \chi(2\Delta_{\bar{p}} - 2\Delta_p + \xi)}{\mathcal{N}_3^{[p]}(\xi)} \tilde{a}_{\text{in}}[2\Delta_{\bar{p}} - 2\Delta_p + \xi] + \right.$$

Two-mode squeezing:

$$\langle \tilde{a}_{\text{out}}[\xi]\tilde{a}_{\text{out}}[2\Delta_2 - \xi']\rangle = \tfrac{1}{2}\exp(i\varphi_2)\sin\theta\sinh 2\lambda \times \delta(\xi - \xi')$$

“Beam splitter correlations”:

$$\left\langle (\tilde{a}_{\text{out}}[2\Delta_1 - \xi])^\dagger \tilde{a}_{\text{out}}[2\Delta_2 - \xi'] \right\rangle = \frac{\sin 2\theta}{2} e^{i(\varphi_2 - \varphi_1)} \sinh^2\lambda \times \delta(\xi - \xi')$$

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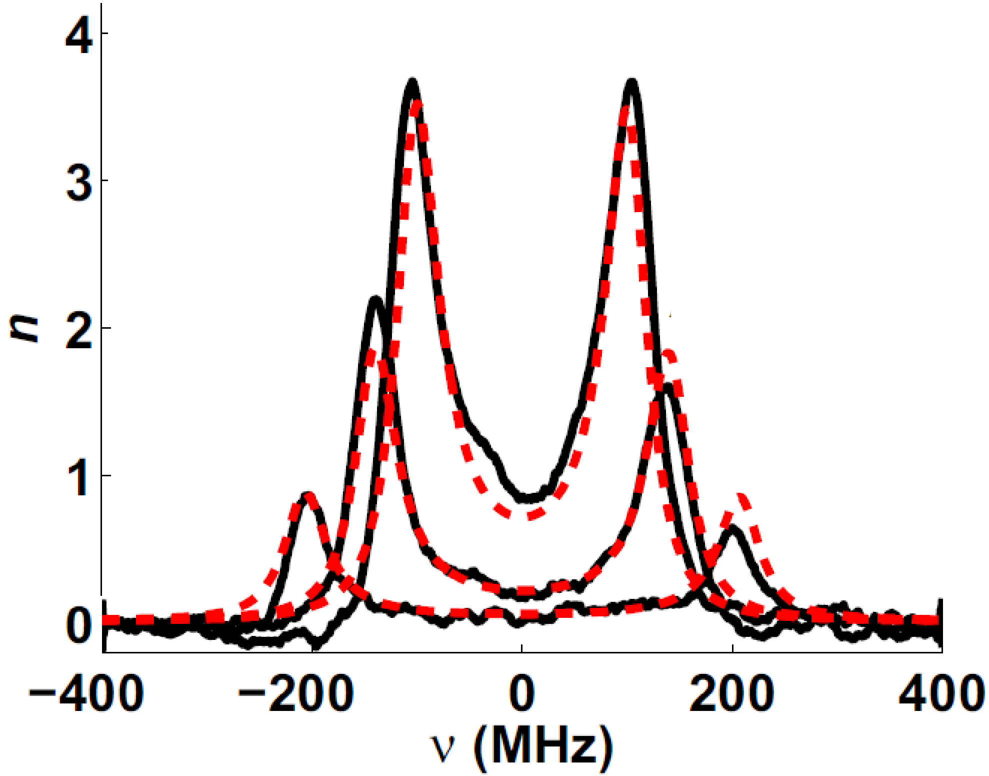
# Peaks at fixed detuning




$$\langle \tilde{a}^{\dagger}_{\text{out}}(-\nu)\,\tilde{a}_{\text{out}}(\nu')\rangle$$

- seen only in the vicinity of the cavity resonance

- squeezing correlations:

$$\langle \tilde{a}_{\text{out}} \tilde{a}_{\text{out}} \rangle_{T=0}(\nu)$$

NIST, Chalmers, NEC
ETH, Paris, Yale, …



Lähteenmäki, P., Paraoanu, G. S., Hassel, J. & Hakonen, P. J.
Proc. Natl. Acad. Sci. **110**, 4234–4238 (2013).

# Displacement operator

**Coherent states can be generated using the displacement operator:**

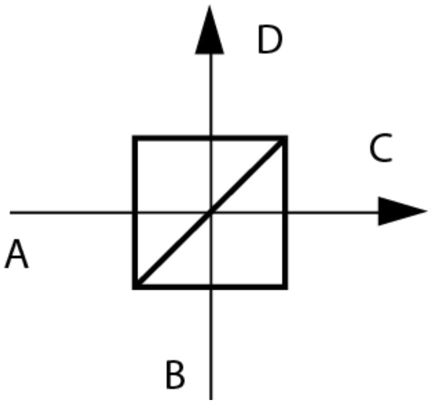
$$D(\alpha) = \exp\left(\alpha a^\dagger - \alpha^* a\right) \qquad D(\alpha) = e^{-\frac{1}{2}|\alpha|^2} e^{-\alpha a^\dagger} e^{-\alpha^* a}$$

$$|\alpha\rangle = D(\alpha)|0\rangle$$

**Glauber state** $$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

**Minimum uncertainty, equal between *$X_1$* and *$X_2$*** $$\Delta X_1 = \Delta X_2 = \frac{1}{\sqrt{2}}$$

# Beam splitter




$$\begin{pmatrix} a_{k,C} \\ a_{k,D} \end{pmatrix} = U \begin{pmatrix} a_{k,A} \\ a_{k,B} \end{pmatrix}$$

$$U = \begin{pmatrix} \cos\theta & e^{i\varphi}\sin\theta \\ -e^{-i\varphi}\sin\theta & \cos\theta \end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

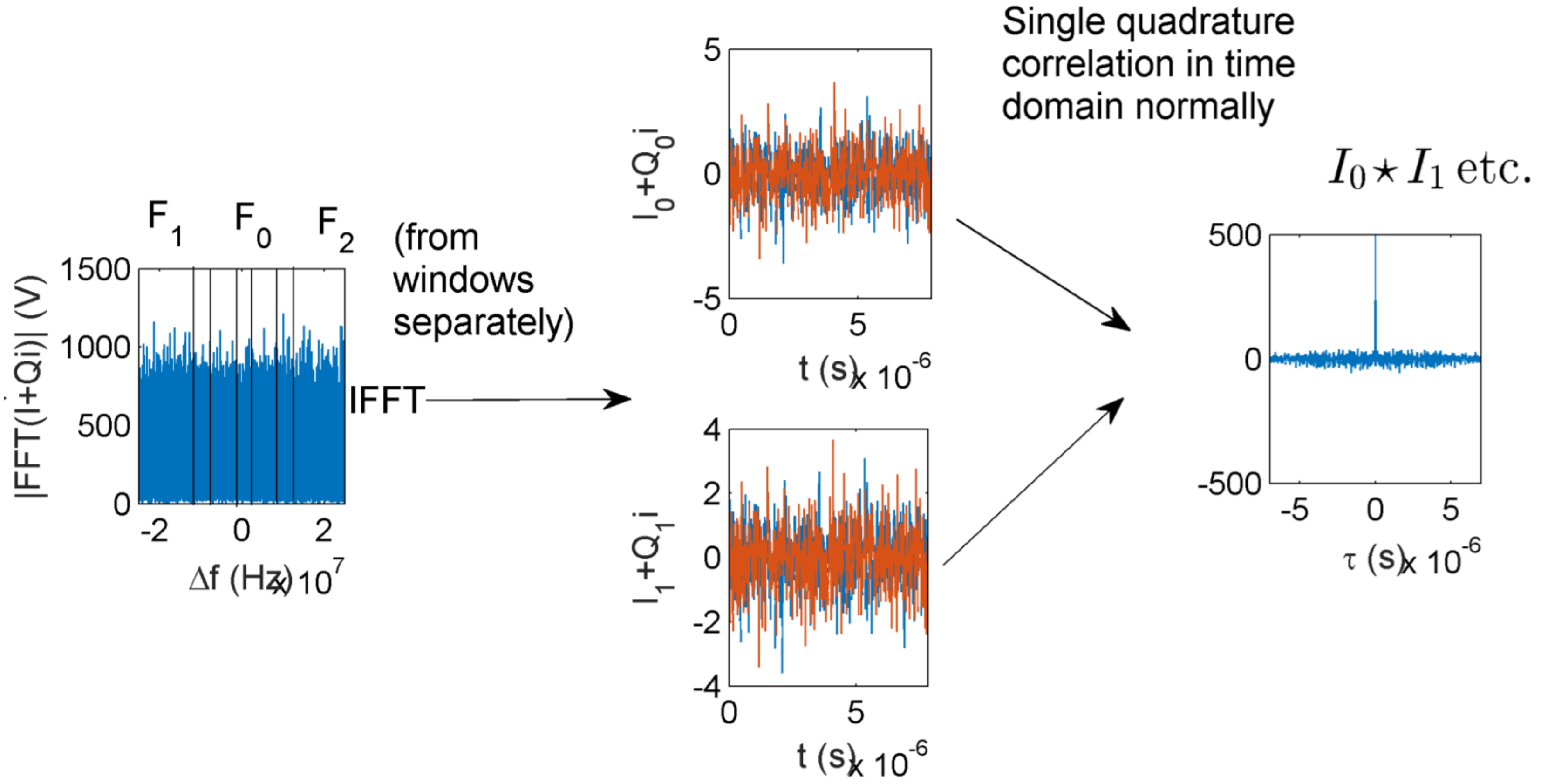
$$\begin{aligned} a_C^\dagger a_C &= (\cos\theta a^\dagger{}_A - \sin\theta a^\dagger{}_B)(\cos\theta a_A - \sin\theta a_B) \\ &= \cos^2\theta a^\dagger{}_A a_A + \sin^2\theta a^\dagger{}_B a_B - \sin\theta\cos\theta(a^\dagger{}_A a_B + a^\dagger{}_B a_A) \end{aligned}$$

$$\left\langle a_C^\dagger a_C \right\rangle = \left\langle a_D^\dagger a_D \right\rangle = 1$$

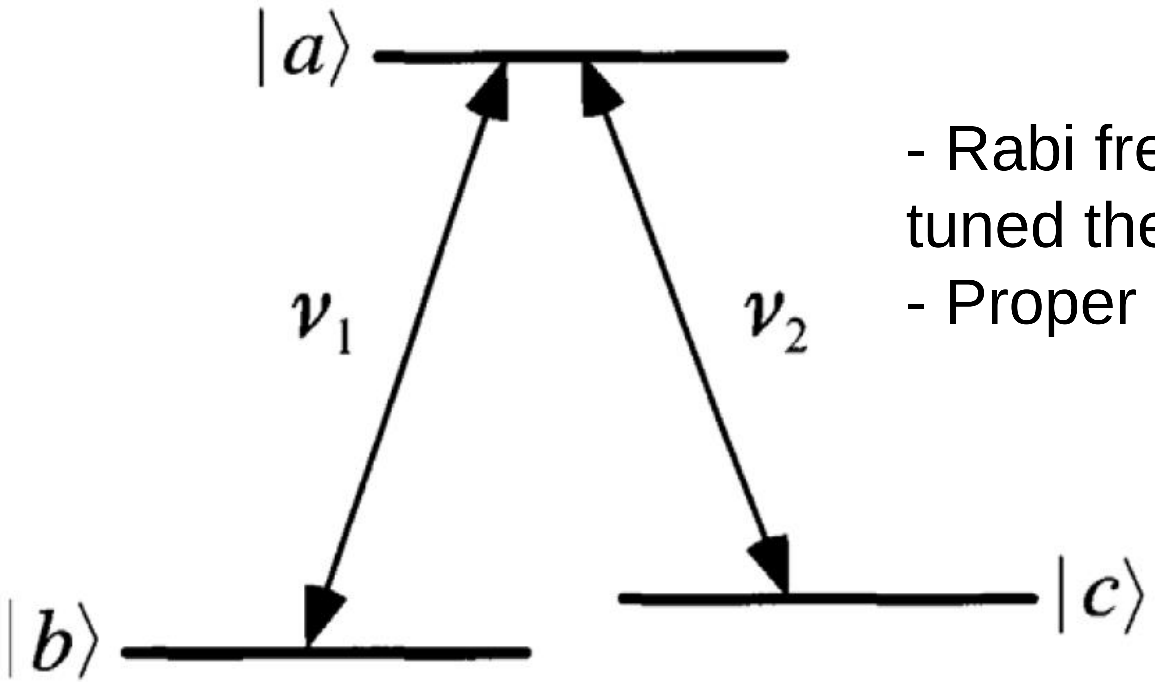
$$\begin{aligned} a_D a_C &= (\sin\theta a_A + \cos\theta a_B)(\cos\theta a_A - \sin\theta a_B) \\ &= (\cos^2\theta - \sin^2\theta) a_A a_B + O(a_A^2, a_B^2) \end{aligned}$$

$$\left\langle a_C^\dagger a_D^\dagger a_D a_C \right\rangle = \cos^2 2\theta.$$

# Data analysis II



# Coherent population trapping (CPT)




- Rabi frequencies tuned the same
- Proper phases

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}|b\rangle + e^{-i\varphi}\frac{1}{\sqrt{2}}|c\rangle$$

Dark state:
- population trapped on $|b\rangle$ & $|c\rangle$
- no absorption