# Non-equilibrium noise in a simple model of glassy dynamics

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## Content of the talk:

1. The model of one-dimensional potential with short-range correlated random force: formulation and possible physical origin
2. Basic parameters and regimes
3. Sub-diffusion regime $\kappa = T/E_0 < 1$ :
   non-stationary distribution of escape times.
4. *ac* response and noise within dynamically self-induced well :
   simple aging effect

# Diffusion in a random-field-induced potential

$$\frac{dx}{dt} = f(x) + F + \eta = -\frac{\partial U}{\partial x} + \eta$$

$$\langle f(x) \rangle = 0, \ \langle f(x) f(x') \rangle = \gamma \, \delta(x - x')$$

Static random force

$F$ is the driving force

$\eta$ is the thermal noise: $\langle \eta(t) \eta(t') \rangle_T = 2 T \delta(t - t')$.


TEMKIN, D. E., *Dokl. Akad. Nauk SSSR* **206** (1972) 27.

KESTEN, H., KOZLOV, M. and SPITZER, F., *Compos. Math.* **30** (1975) 145.

BOUCHAUD ET AL. ANNALS OF PHYSICS **201**, 285–341 (1990)

SINAY, Ya. G., *Theor. Probab. Its Appl.* **27** (1982) 247.

DERRIDA, B. and POMEAU, Y., *Phys. Rev. Lett.* **48** (1982) 627.

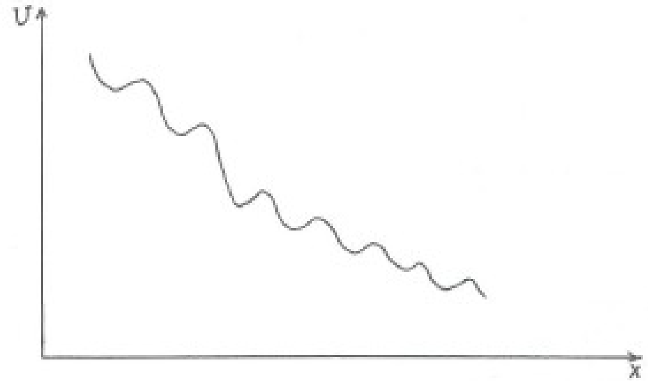
BOUCHAUD, J. P., GEORGES, A., LE DOUSSAL, P., *J. Phys. France* **48** (1987) 1855.


Realizations:

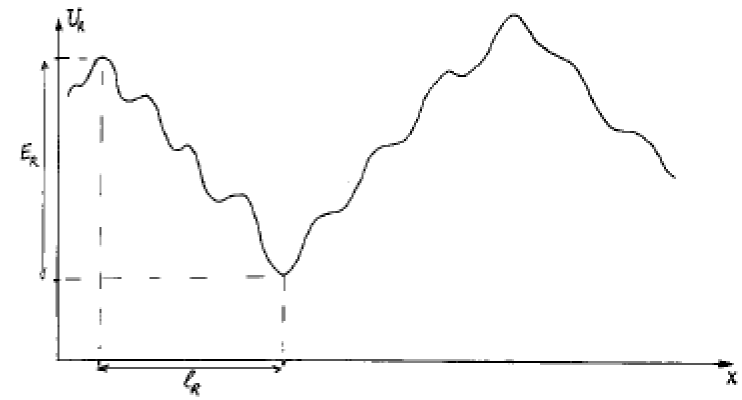
Kink in the 1D Random-Field Ising model
Phase boundary between 2 phases near disordered 1st-order transition
A particle coupled to an linear defect in 2D disordered media

# Basic parameters and regimes

Typical “bare” energy relief




Typical relief around moving particle




$\tau \sim \exp(E/T)$ is the delay time

Probability to find high energy barrier

$$p(E) \simeq E_0^{-1} \exp(-E/E_0), \quad E \gg E_0$$

where $E_0 = \frac{\gamma}{2F}$

The corresponding length is

$$x_0 = \frac{\gamma}{4F^2}$$

The delay time distribution $\Psi(\tau) \simeq \frac{1}{\tau_0}\left(\frac{\tau_0}{\tau}\right)^{1+\kappa}, \quad \kappa = \frac{T}{E_0}$

is valid for $\tau \gg \tau_1 = \tau_0 \exp(1/\kappa)$

# Derivation of the ρ(E) distribution

Random potential relief ε(x) is defined by a random walk

$$\frac{d\epsilon}{dx} = -F + f(x) = -\frac{\partial V(\epsilon)}{\partial \epsilon} + f(x) \qquad \langle f(x) f(x') \rangle = \gamma \delta(x - x')$$

This RW can be considered like a Langevin equation
in presence of noise (x→ t) and potential :

$$V(\epsilon) = \epsilon F$$

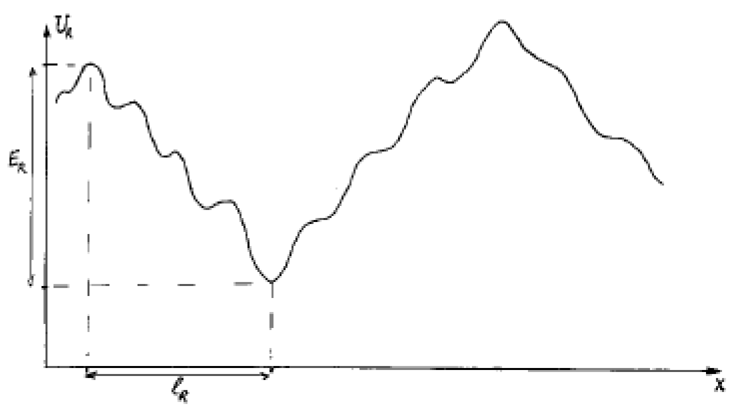
Effective temperature:

$$\frac{1}{2}\gamma \equiv T^{\mathrm{eff}}$$

$$P(\epsilon) \propto \exp\left(-\frac{V(\epsilon)}{T^{\mathrm{eff}}}\right) \propto \exp\left(-\frac{\epsilon}{E_0}\right)$$

where $E_0 = \dfrac{\gamma}{2F}$

Probability to find high energy barrier



Consider first the high-temperature region, $\kappa > 1$. The mean total time which the particle needs to get through the system of length $L$ is

$$t_{\text{tot}} = \frac{L}{x_0} \int_0^\infty \tau \Psi(\tau)\, \mathrm{d}\tau \simeq \tau_0 \frac{L/x_0}{\kappa - 1}$$

$$\Rightarrow \quad \langle x \rangle \simeq vt\,, \quad v = (\kappa - 1)\frac{x_0}{\tau_0}$$

average velocity > 0

Dynamic phase transition: when $\kappa = \dfrac{T}{E_0}$ goes to unity, $v \to 0$

At $\kappa < 1$ the average time to travel the segment of length L **diverges**

That indicates the lack of self-averaging among the energy barriers:
very rare but also very deep potential wells dominate statistics

$\tau_{\max}(L)$ is the largest probable waiting time $\Rightarrow$

$$\frac{L}{x_0} \int_{\tau_{\max}(L)}^\infty \Psi(\tau)\, \mathrm{d}\tau \simeq 1$$

$$\tau_{\max}(L) \simeq \tau_0 (L/x_0)^{1/\kappa}\,, \quad \kappa < 1\,. \quad \Rightarrow \quad x \sim t^\kappa\,, \quad \kappa < 1$$

# Non-stationary drift at $\kappa < 1$

The origin of sub-linear drift can be understood as a result of subsequent trapping of a particle by more and more deep potential wells as the total time of motion increases: this is an example of **aging effect**

New notion will be useful: ***renormalized*** distribution of time delays

$$\Psi_R(t) \sim t\Psi(t) \sim t^{-\kappa} \quad (\tau_1 \lesssim t \lesssim \tau_{max}(L))$$

Probability density to find a particle in a well with delay time t after it moves for the time t

The corresponding renormalized distribution for energy barriers:

$$p_R(E) \sim \exp\left(\frac{1-\kappa}{T}E\right)$$

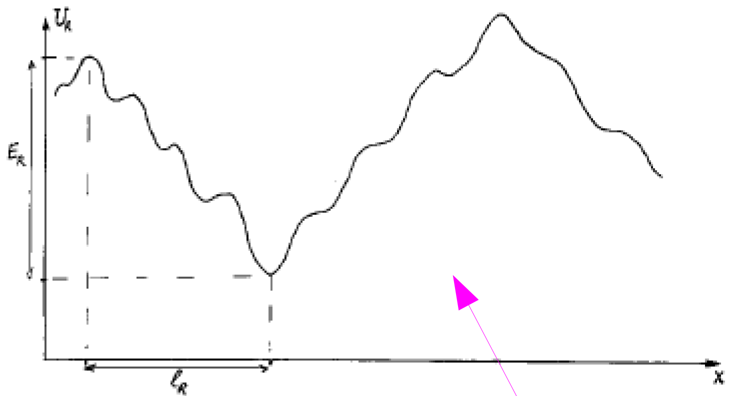
at $\kappa > 1$ $\quad \langle E_R \rangle \simeq \frac{T}{\kappa - 1}$

Stationary regime sets in at

$$t_w \gg t_R \sim e^{1/(\kappa-1)},$$

The barrier size $\quad \langle \ell_R \rangle \simeq \frac{\langle E_R \rangle}{F} \simeq \frac{T}{(\kappa-1)F}$

Typical relief around moving particle




$$\langle \ell_R \rangle \simeq \frac{\langle E_R \rangle}{F} \simeq \frac{T}{(\kappa - 1) F} \qquad \text{at } \kappa > 1$$

$$l_R \to \infty \quad \text{as} \quad t_w \to \infty \qquad \text{at } \kappa < 1$$

Non-stationary time-delay distribution at $\kappa < 1$ :

$$\Psi_R(\tau \,|\, t_w) \simeq \frac{1-\kappa}{t_w^{1-\kappa}} \tau^{-\kappa}, \quad \tau_1 \leq \tau \leq t_w$$

$$\langle E_R \rangle \simeq T \ln t_w, \quad \langle \ell_R \rangle \simeq (T/F) \ln t_w$$

On time-scales $t \ll t_w$ particle can be considered to sit inside symmetric potential well of the form

$$U_R(x) = F\,|x| + \int_0^x f(x')\,dx'$$

# $ac$ response and noise within dynamically self-induced potential well

(in the limit of infinite waiting time $t_w$)

Distribution function P(x,t) determines probability density to find a particle near the point $x$ at time $t$ in presence of an oscillating force $f_\omega \, e^{-i\omega t}$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\left[ T \frac{\partial P}{\partial x} + \left( \frac{\partial U_R}{\partial x} - f_\omega \, e^{-i\omega t} \right) P \right]$$

$$P(x,t) = P_0(x) + P_1(x)\, f_\omega \, e^{-i\omega t}$$ where $P_0(x) = \exp[-U_R(x)/T]$

is the equilibrium solution inside renormalized potential well

susceptibility $\chi(\omega) \equiv \partial \langle x(\omega) \rangle / \partial f_\omega = \int dx \, P_1(x)\, x$

can be expressed in terms of eigenfunctions $|\alpha\rangle$ and eigenvalues $\varepsilon_\alpha$

of the Schrodinger – like equation $T \frac{\partial \psi}{\partial t} = T^2 \frac{\partial^2 \psi}{\partial x^2} - \left[ \frac{1}{4} \left( \frac{\partial U_R}{\partial x} \right)^2 - \frac{T}{2} \frac{\partial^2 U_R}{\partial x^2} \right] \psi$

# Relation between *Fokker-Planck* and ( imaginary-time) *Schrödinger* equations

Fokker-Planck equation for probability density:

$$\dot{\varphi}_x = -\Gamma \frac{\delta V}{\delta \varphi_x} + \xi(x,t)$$

$$\langle \xi(x,t)\,\xi(x',t')\rangle = 2T\Gamma\,\delta(x-x')\,\delta(t-t')$$

$$\frac{1}{\Gamma}\frac{\partial \mathscr{P}}{\partial t} = \int_{(x)} \frac{\delta}{\delta\varphi_x}\left(T\frac{\delta\mathscr{P}}{\delta\varphi_x} + \frac{\delta V}{\delta\varphi_x}\mathscr{P}\right)$$

Stationary solution:

$$\mathscr{P}\{\varphi_x\} = A^{-1}\exp\left(-\frac{1}{T}V\{\varphi_x\}\right)$$

under the condition

$$A = \int \exp\left(-\frac{1}{T}V\{\varphi_x\}\right) D\varphi_x < \infty$$

Transformation

$$\mathscr{P}\{\varphi_x\} = \exp\left(-\frac{1}{2T}V\{\varphi_x\}\right)\Psi\{\varphi_x\}$$

leads to

$$\frac{T}{\Gamma}\frac{\partial}{\partial t}\Psi = \int_x \left(T^2\frac{\delta^2\Psi}{\delta\varphi_x^2} - U\{\varphi_x\}\Psi\right),$$

where

$$U\{\varphi_x\} = \int_{(x)}\left[\frac{1}{4}\left(\frac{\delta V}{\delta\varphi_x}\right)^2 - \frac{T}{2}\frac{\delta^2 V}{\delta\varphi_x^2}\right]$$

Susceptibility in terms of the Schrödinger eigenfunctions :

$$\chi(\omega) = \frac{1}{T}\sum_{\alpha} \frac{\varepsilon_\alpha}{\varepsilon_\alpha - i\omega T} \,|\langle 0|x|\alpha\rangle|^2 .$$

$$\langle 0|x^2|0\rangle = \frac{2T^2}{F^2} \equiv x_T^2$$

Note that $\chi(0) = \frac{1}{T}\langle 0|x^2|0\rangle$ as it should be

Introducing spectral density $g(\varepsilon) = \sum_{\alpha} \delta(\varepsilon - \varepsilon_\alpha) |\langle 0|x|\alpha\rangle|^2$ one obtains

$$\chi(\omega) = \frac{1}{T}\int \frac{d\varepsilon\, g(\varepsilon)\, \varepsilon}{\varepsilon - i\omega T} .$$

$g(\varepsilon)$ is well-defined for renormalized potential $U_R(x)$ only

It is possible to show that at low energies $\varepsilon \ll 1/\tau_1$

$$\tau_1 = \tau_0 \exp(1/\kappa)$$

$$g(\varepsilon) \propto \rho(\varepsilon) = \sum_{\alpha} \delta(\varepsilon - \varepsilon_\alpha) \propto \varepsilon^{\kappa - 1}$$

Localization length does not depend on energy $\varepsilon$ : $l_\varepsilon = T/F$

Bouchaud, J. P., Comtet, A., Georges, A. and Le Doussal, P., *Europhys. Lett.* 3 (1987) 653.

(for bare potential U(x) )

# Dissipative response and noise in $t_w \to \infty$ limit

$$\chi(\omega) = \frac{1}{T}\int \frac{d\varepsilon\, g(\varepsilon)\,\varepsilon}{\varepsilon - i\omega T}\,. \quad \Rightarrow \quad \Im\chi(\omega) = \frac{2T}{F^2}\int d\epsilon\rho(\epsilon)\frac{\epsilon\,\omega T}{\epsilon^2 + (\omega T)^2}$$

$$\langle 0|x^2|0\rangle = \frac{2T^2}{F^2} \equiv x_T^2$$

at small $\kappa \ll 1$

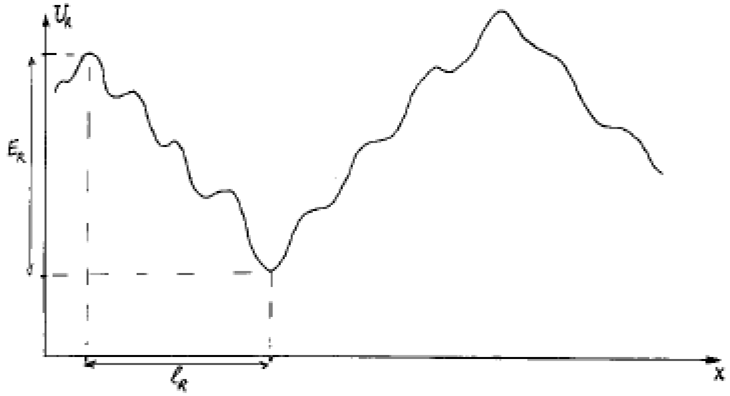
$$\Im\chi(\omega) = \# \frac{T^2}{\gamma F}\left(\frac{T^2}{\gamma x_0}\right)^{2\kappa} \omega^\kappa \approx \omega^\kappa x_T^2 \frac{F}{\gamma}$$

Noise power:

$$C(\omega) = \langle x_\omega^2 \rangle = \frac{2T}{\omega}\Im\chi(\omega) \approx \frac{T}{\omega^{1-\kappa}}\, x_T^2\, \frac{F}{\gamma}$$

Nearly 1/f noise at low values of $\kappa$

# Dissipative response and noise at finite long $t_w$




$$\langle E_R \rangle \simeq T \ln t_w , \quad \langle \ell_R \rangle \simeq (T/F) \ln t_w$$

Distribution of the delay times:

$$\Psi_R(\tau \,|\, t_w) \simeq \frac{1-\kappa}{t_w^{1-\kappa}} \tau^{-\kappa} , \quad \tau_1 \lesssim \tau \lesssim t_w$$

The eigenstates within a well are now quasi-discrete, with a nonzero decay rates

$\Gamma_\alpha$ are of the order of $\tau^{-1} \simeq \exp(-E/T)$ mean decay rate $\Gamma = \langle \Gamma_\alpha \rangle_R$ is

$$\Gamma = \int \frac{d\tau}{\tau} \Psi_R(\tau \,|\, t_w) \simeq \frac{1-\kappa}{\kappa} t_w^{\kappa - 1}$$

is much larger than $1/t_w$

$$\mathfrak{Im}\, \chi(\omega) \propto \omega^\kappa f\left( \frac{1}{\omega t_w^{1-\kappa}} \right)$$

$$C(\omega) \propto \frac{T}{\omega^{1-\kappa}} f\left( \frac{1}{\omega t_w^{1-\kappa}} \right)$$

$f(0) = 1$

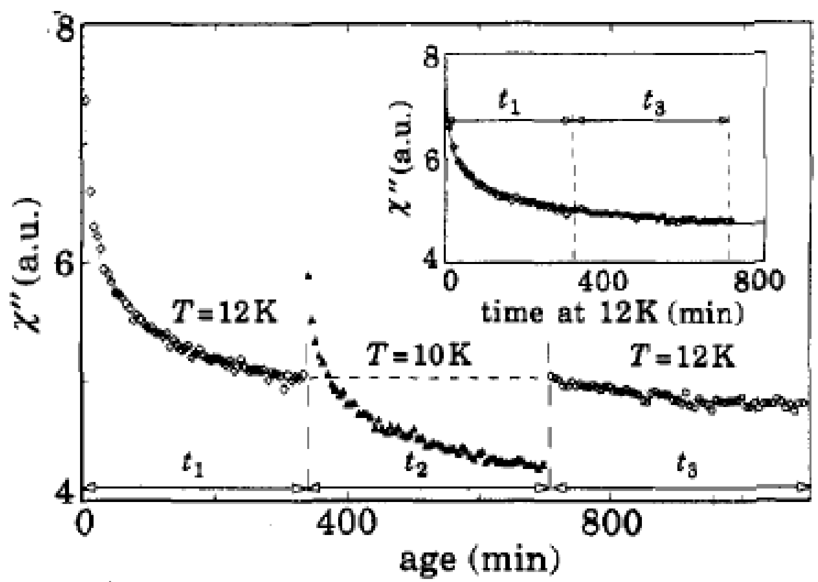
*Non-stationary "1/f" noise*

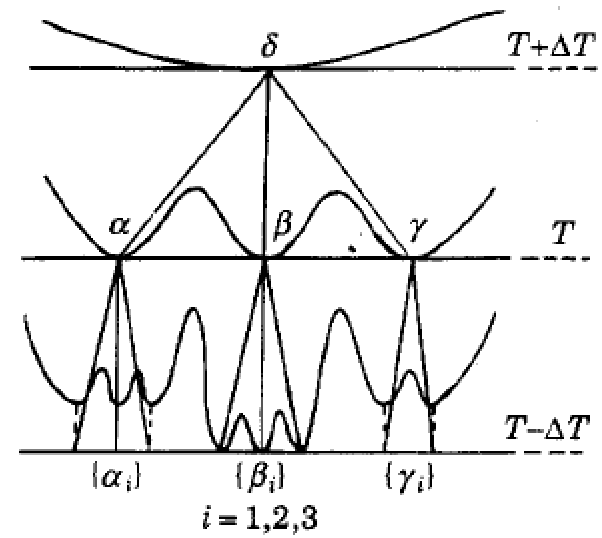
# Aging in spin glasses

*Europhys. Lett.*, 18 (7), pp. 647-652 (1992)

F. LEFLOCH, J. HAMMANN, M. OCIO and E. VINCENT

**Can Aging Phenomena Discriminate between the Droplet Model and a Hierarchical Description in Spin Glasses?**




Evolution with age of $\chi''$ during a temperature cycle for $\Delta T = 2$ K and $t_1 = t_2 = t_3 = 350$ min

$\omega/2\pi = 0.1$ Hz

$$\sigma = \sigma_0 \lambda^{-\alpha} \exp[-(\lambda/\tau)^{\beta}], \quad \tau \propto t_w^{\mu}.$$

$$f \ll f_0(T), \qquad \chi''(f) \simeq (f/f_0)^{\alpha(T)} \chi''(f_0),$$

$$f \gg f_0(T), \qquad \chi''(f) \simeq (f/f_0)^{-\beta(T)} \chi''(f_0)$$

$$\sigma = \sigma_0 \lambda^{-\alpha} \exp[-(\lambda/\tau)^{\beta}], \quad \tau \propto t_w^{\mu}.$$

# Conclusions

- Simple 1D model accounts for dynamically generated hierarchy of relaxation modes and aging behavior
- On moderately high frequency scales, $1/\omega^{\mu}$ noise is obtained, with $1-\mu = \kappa << 1$ at low temperatures
- For a finite $\omega t_w$ a correction to a pure power law depends on the scaling parameter $\omega t_w^{\mu}$ in agreement with experiments on spin glasses (M.Ocio et al)