NAME AND SURNAME

TEST II *January 22nd 2014*

(Ex 1) *(2 pts)* Calculate the area of region $R = \{(x, y) : e^x \leq y \leq 3^x, 0 \leq x \leq 2\}$ using a double integral. Draw a suitable graph.

(Ex 2) *(2 pts)* Calculate $\iint_D (x - y)^2 dxdy$ if $D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, y \geq |x|\}$. Use polar coordinates.

(Ex 3) *(1 pt)* Draw the surface $z = -\sqrt{4 - x^2 - y^2} + 1$ and describe it in your own words.

(Ex 4) *(1+1 pts)*

a) Find the mass of a parallelepiped $R = [0, 1] \times [0, 2] \times [0, \pi]$ with density $\rho(x, y, z) = (x + y)\sin(z)$.

b) Draw the parallelepiped and discuss its density at two different points and along the z-axis.

(Theory 1) *(2×1 pt)* Consider calculating volume of a solid based on a region $D = [a, b] \times [c, d]$ bounded from above by the graph of $f(x, y)$ and from below by the OXY-plane. How would you calculate the volume using a) a double integral, b) a triple integral? Write down appropriate formulas.

(Theory 2) *(1+2 pts)* a) Give the definition of spherical coordinates, mark all angles and the radius on a diagram.

x =

y =

z =

b) Consider an upper hemisphere with radius 2 out of which an upper part of a cone was cut out. The cone was slanting at the radius of $\frac{\pi}{4}$ with respect to the OXY-plane. Describe the remaining part of the hemisphere in spherical coordinates and calculate its volume.

(Ex 5) *(2 pts)* Find y_g and y_p for $y'' = x^2$ if $y(0) = 1$, $y(1) = 2$.

Ex 6) *(2 pts)* Find y_g using separation of variables for $\frac{xy}{x+1} = \frac{dx}{dy}$,

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(Ex 7) *(2 pts)* Find y_g in a linear equation: $(x^2 - 1)y' + 2xy = x$.

(Ex 8) *(2×1 pt)* a) Find y_g for $y'' + 6y' + 13y = 0$

b) "Guess" y_p for $y'' + 6y' + 13y = x^2$ but do not calculate any constants.