| NAME AND SURNAME | INDEX NUMBER | GROUP |
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TEST II *January 17th*

**(Ex 1)** *(3 pt)* Calculate the area of the lamina $R = \{\cos x \leq y \leq \sin x, x \in [\frac{\pi}{4}, \frac{5\pi}{4}]\}$ using a double integral. Draw a suitable graph.

**(Ex 2)** *(3 pt)* Calculate $\iint\limits_D \sqrt[3]{x^2+y^2}dxdy$ if $D = \{4 \leq x^2 + y^2 \leq 9, x \geq 0, y \leq 0\}$. Use polar coordinates.

**(Ex 3)** *(1+1 pts)* Draw the following surfaces and describe them in your own words:

a) $z = \sqrt{x^2+y^2} - 2$

b) $z = -x^2 - y^2 - 1$

**(Ex 4)** *(2+1 pts)* a) Find the mass of a parallelepiped $R = [0,2] \times [0,3] \times [0,4]$ if $\rho(x,y,z) = x \cdot y \cdot z$.

b) Discuss density at three different points. What is the density along the $OY$ axis?

**(Ex 5)** *(2 pts)* Check if $y = \frac{x^3}{2} + 2x$ is a solution to the equation $y' - \frac{y}{x} = x^2$.

**(Ex 6)** *(3 pts)* Find $y_g$ and $y_p$ using the method of direct integration. $y'' = \sin x - \cos x$, $y(0) = 1$, $y(\frac{\pi}{2}) = 1$.

**Ex 7)** *(2 pts)* Find $y_g$ using separation of variables. $y' = (x+1)^2 y^2$,

**(Ex 8)** *(2 pt)* Find $y_g$ in a linear equation: $y' + y \cdot \frac{3}{x} = x$.

**(Ex 9)** *(2 pts)* **Bonus exercise:** Calculate the center of mass of a square $D = [0,1] \times [0,1]$ having density $\rho(x,y) = xy + 1$.