NAME AND SURNAME	INDEX NUMBER	GROUP

TEST II *January 17th*

(Ex 1) *(3 pt)* Calculate the area of the lamina $R = \{x^2 \leq y \leq x, x \in [0,1]\}$ using a double integral. Draw a suitable graph.

(Ex 2) *(3 pt)* Calculate $\iint\limits_D \frac{x}{x^2+y^2} dxdy$ if $D = \{x^2 + y^2 \leq 16, y \leq -x\}$. Use polar coordinates.

(Ex 3) *(1+1 pts)* Draw the following surfaces and describe them in your own words:

a) $z = -\sqrt{1 - x^2 - y^2} + 1$

b) $z = -\sqrt{x^2 + y^2} - 1$

(Ex 4) *(2+1 pts)* a) Find the mass of a parallelepiped $R = [0,1] \times [0,2] \times [0,3]$ if $\rho(x,y,z) = x \cdot y^2 \cdot \frac{z}{2}$.

b) Discuss density at three different points. What is the density along the OX axis?

(Ex 5) *(2 pts)* Check if $y = -x + x^3$ is a solution to the equation $xy' - 3y = 2x$.

(Ex 6) *(3 pts)* Find y_g and y_p using the method of direct integration. $y'' = 3x^2 + 4x + 5$, $y(0) = 0$, $y(1) = 1$.

Ex 7) *(2 pts)* Find y_g using separation of variables. $y' = -y^2 e^x$,

(Ex 8) *(2 pt)* Find y_g in a linear equation: $y' + 2xy = xe^{-x^2}$.

(Ex 9) *(2 pts)* **Bonus exercise:** Calculate the center of mass of a square $D = [0, 1] \times [0, 1]$ having density $\rho(x, y) = xy + 1$.