NAME AND SURNAME	INDEX NUMBER	GROUP

TEST II *January 20th*

Ex 1. Calculate the integral $\int_0^1 \int_{x^2}^{\sqrt{x}} xy \, \mathrm{dx}\, \mathrm{dy}$ and draw the region over which it is calculated.

Ex IX.3.b. Compute the area of the region bounded by curves $y = e^x, y = e^{3x}, x = 2$ using a double integral. Draw the area.

Ex IX.1.b Calculate $\iint_D \frac{\mathrm{dx}\, \mathrm{dy}}{\sqrt{x^2+y^2}}$, where D is a ring between circles $x^2+y^2 = 1$ and $x^2+y^2 = 4$. Use polar coordinates.

Ex 4. Consider calculating volume of a solid based on region $D = [a, b] \times [c, d]$, bounded from above by a function $f(x, y)$ and from below by the OXY plane. How would you calculate that volume using a) a double integral, b) a triple integral? Write down the integrals.

Ex 5. Consider an upper hemisphere with radius $r = 1$, out of which an upper cone was cut out. The cone creates an angle of $\psi = \frac{\pi}{3}$ with the OXY plane. Draw the remaining part of the upper hemisphere for $x \geq 0, y \leq 0$ and calculate its volume using spherical coordinates.

Ex 6. Find y_g and y_p using the method of direct integration if $y'' = 3x^2 + 2x + 1$, $y(1) = 0$ and $y(2) = 1$.

Ex 7. Find y_g using separation of variables if $y' = \frac{xy}{x+1}$.

Ex 8. Solve a linear equation $y' + y = e^x$ using <u>two different methods</u>.

(Bonus) Find y_g if $y' = \frac{x+y}{x}$. Use a method appropriate for homogenous functions.