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**Ex 1.** (2 pts.) Using the sequence of partial sums  $S_n$  establish if the series  $\sum_{n=1}^{\infty} \cos(n\pi)$  converges or diverges.

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**Ex 2.** (1 pt.) a) Give a definition of a comparison test.

(2 pts.) b) Find if the series  $\sum_{n=1}^{\infty} \frac{n+1}{2n^2-1}$  diverges or converges using any appropriate method.

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**Ex 3.** (3 pts.) Establish the type of convergence (divergent, absolutely convergent, conditionally convergent) of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$ . (Justify each condition of the alternating series test using at least one sentence).

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**Ex 4.** (2 pts.) a) Calculate the limit  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}$ .

(2 pts.) b) Show that the limit  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$  does not exist using any method you wish.

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**Ex 5.** (2 pts.) Find all first and second derivatives of  $f(x, y) = \frac{x}{y} + \frac{y}{x}$ .

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**Ex 6.** (4 pts.) Find all extreme values of  $f(x, y) = \frac{1}{5x} + \frac{5}{y} - xy$ .

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**Ex 7.** (2 pts.) Calculate the approximated value and all errors of  $\sqrt{(3.2)^2 + (3.9)^2}$ .

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**Bonus** (2 pts.) Give an equation of a plane  $\Pi$  that is tangent to  $f(x, y) = \ln(2x + y)$  at  $P = (-1, 3, 0)$ .