

Ex. 1 (2+3 pts) a) Give the definition of the sum of the series $\sum_{n=1}^{\infty} a_n$.

b) Use partial fraction expansion to find the sum of $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$.

Ex. 2 (2+3+2 pts) Determine if the following series are convergent or divergent: a) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$, b) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$, c) $\sum_{n=1}^{\infty} \frac{n}{3^n}$.

Ex. 3 (3+4 pts) a) Evaluate the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^4}{x - y}$.

b) Verify the existence of limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$.

Ex. 4 (4 pts) Find all partial derivatives of the 2nd degree of $f(x, y) = \ln(x^2 + y^2)$.

Ex. 5 (6 pts) Find extreme values of the function $f(x, y) = e^y(x^2 + y)$.

Ex. 6 (4+5 pts) a) Evaluate $\iint_D \frac{xdy}{\sqrt{x^2+y^2}}$ if D is a part of the ring between circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the 1st quadrant.

b) Evaluate $\iiint_D (x^2 + y^2 + z^2) dx dy dz$, where $D = \{(x, y, z) : -\sqrt{4 - x^2 - y^2} \leq z \leq 0\}$.

Ex. 7 (3+4+5 pts) Find general solutions of the following equations:

a) $y' = -y^2 e^x$ (separate the variables), b) $y' - xy = x e^{x^2}$ (use the integrating factor),

c) $y'' - 4y' + 4y = e^{2x}$ (use the method of undetermined coefficients).