

Ex 1. (a) (1pt) What is the necessary condition for convergence of the infinite series $\sum_{n=1}^{\infty} a_n$?

(b) (2pts) Give two examples of infinite series, (i) that doesn't satisfy the necessary condition, and (ii) that satisfies the necessary condition but is divergent.

(i)

(ii)

(c) (3pts) Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{3^n} - \frac{1}{3^{n+1}} \right)$ using the sequence of partial sums.

Ex 2. Establish the convergence of the series using an appropriate test

(a) (2pts) $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

(b) (2pts) $\sum_{n=1}^{\infty} \left(\frac{4n^2 - 3}{7n^2 - 6} \right)^n$

Ex 3. (4pts) Show that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{y}{\sqrt{x^2 - y^2}}$ does not exist.

Ex 4. (5pts) Evaluate the integral $\iint_D \frac{xe^{2y}}{4-y} dx dy$, where D is the region bounded by the lines $x = 0$, $x = 2$, $y = 0$, and the curve $y = 4 - x^2$.

(Tip: choose the order of integration wisely)

Ex 5. (5pts) Find the volume of the solid bounded by $z = \sqrt{8 - x^2 - y^2}$ (from above) and $z = \sqrt{x^2 + y^2}$ (from below).

Ex 6. (5pts) Using the total differential approximate the value of the expression: $\ln(0.95) + \frac{1}{\sqrt[3]{8.1}}$

Ex 7. (6pts) Find all local extreme values and saddle points of $f(x, y) = x^3 - y^3 + 3xy$.

Ex 8. Find the general solutions of the following equations

(a) (4pts) $y' - y \sin x = 2xe^{-\cos x}$

(b) (5pts) $y'' + 2y' - 3y = 3x + 4$

Ex 9. (a) (1pt) What is an *equilibrium solution* of an autonomous differential equation $y'(x) = f(y)$?

(b) (3pts) Find all equilibrium solutions of $y' = (y^2 - 1)(y - 2)$ and classify them as asymptotically stable, unstable, or semistable.

Ex 10. (2pts) In the movie "Flatland" Spherius comes to reveal the secret to the Flatlanders, the secret that is also hidden in the area 33h. What is the secret ?