FINAL EXAM IN MATHEMATICS FOR EPM III - JANUARY 30TH, 2013

**Ex. 1** *(2+3 pts)* a) Give the definition of the sum of the series $\sum_{n=1}^{\infty} a_n$.

b) Calculate the sum of $\sum_{n=1}^{\infty} (\frac{1}{n^3} - \frac{1}{(n+1)^3})$ directly from the definition.

**Ex. 2** *(3+2+2 pts)* Determine if the following series are convergent or divergent: a) $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$, b) $\sum_{n=1}^{\infty} \frac{(2n)^2}{n^{2n}}$, c) $\sum_{n=1}^{\infty} \frac{2n-1}{3n+2}$.

**Ex. 3** *(3+4 pts)* a) Show that the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ doesn't exist.

b) Verify whether the function $f(x,y) = e^{-x}\cos y - e^{-y}\cos x$ satisfies the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

**Ex. 4** *(4 pts)* Using the total differential approximate the value of $\sqrt{(3.2)^2 + (3.9)^2}$.

**Ex. 5** *(6 pts)* Find extreme values of the function $f(x,y) = 2xy - 3x^2 - 2y^2 + 10$.

**Ex. 6** *(4+5 pts)* a) Evauate $\iint_D \frac{dxdy}{x^2+y^2+1}$ if D is the 1st quadrant part of the disc with the center (0,0) and radius 2 (use polar coordinates).

b) Evaluate $\iiint_D z\sqrt{x^2+y^2}dxdydz$, where D is a region in space bounded by the surfaces $z = 4$ and $z = x^2 + y^2$ (use cylindrical coordinates).

**Ex. 7** *(3+4+5 pts)* Find general solutions to the following equations:

a) $y' + e^y \sin x = 0$ (separate the variables), b) $y' + \frac{1}{x}y = x^2 - 2$ (use the integrating factor), c) $y'' - 2y' + 2y = 2x$.