NAME AND SURNAME	INDEX NUMBER	GROUP

TEST II *January 23rd*

Ex 1. *(2 pts.)* Calculate the area of the region $R = \{(x, y) : e^x \leq y \leq 3^x, 0 \leq x \leq 2\}$ using a double integral. Draw the region.

Ex 2. *(2 pts)* Calculate $\iint_D (x^2 + y^2 + x - y)dxdy$ if $D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq x\}$. Use polar coordinates.

Ex 3. *(2×1 pts)* Draw the following surfaces and describe them in your own words.

a) $z = -x^2 - y^2 + 2$

b) $x = z^2$

Theory *(2+1 pts)* a) Give the definition of spherical coordinates, mark all angles and the radius on a diagram.

$x =$

$y =$

$z =$

$Jacobian =$

b) Describe an upper hemisphere of radius 2 in spherical coordinates. You may draw it if you wish.

$r \in [\ \dots\ ,\ \dots\ ]$, $\phi \in [\ \dots\ ,\ \dots\ ]$, $\theta \in [\ \dots\ ,\ \dots\ ]$.

Theory *(2 pts)* Give two applications of a double integral other than the one used in **Ex 1.** Provide graphs and formulas as well.

Ex 4. *(3 pts)* Find the mass of a parallelepiped $P = [0,1] \times [0,2] \times [0,2]$ if its density is given by function $\rho(x,y,z) = x^4 + y^3$. Draw the parallelepiped and discuss its density at four different points.

Ex 5. *(1 pt)* Check if $y = \frac{5}{2}x^2 + \frac{1}{2x^2}$ is a solution to equation $y' + \frac{2}{x}y = 10x$.

Ex 6. *(1.5 pts)* Find y_g and y_p for $y'' = x^2 + 2x + 1$ if $y(0) = 1$ and $y(1) = 1$.

Ex 7. *(2 pts)* Find y_g if $y^2 - 2x^2y' = 0$.

Ex 8. *(1.5 pts)* Solve a 1st order linear equation $x^2 \cdot y' - 2xy = 3$.