Exercise. 1 Check, using the definition, the convergence and find the sum of the following series.

a) $\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$, b) $\sum_{n=4}^{\infty} \frac{1}{n^2-5n+6}$, c) $\sum_{n=1}^{\infty} \frac{4}{n^2+n}$, d) $\sum_{n=3}^{\infty} \frac{4n-3}{n-2}$,
e) $\sum_{n=1}^{\infty} (\frac{1}{4})^n$, f) $\sum_{n=1}^{\infty} (\sqrt{3})^{-n}$, g) $\sum_{n=1}^{\infty} e^{1-n}$, h) $\sum_{n=0}^{\infty} \frac{5^n-3^n}{7^n}$,
i) $\sum_{n=1}^{\infty} \frac{1+3^n+4^n}{5^n}$, j) $\sum_{n=1}^{\infty} (\sqrt[n]{n} - \sqrt[n+1]{n+1})$, k) $\sum_{n=1}^{\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$, l) $\sum_{n=1}^{\infty} \ln\frac{n}{n+1}$,

Exercise. 2 Transform the following recurring decimal numbers into vulgar fractions.

a) 0.(3), b) 0.(9), c) 0.(13), d) 1.2(47), e) 0.3(756).

Exercise. 3 Check the convergence of the following series.

a) $\sum_{n=1}^{\infty} \frac{3}{4n+3}$, ą) $\sum_{n=1}^{\infty} \frac{2}{3n-4}$, b) $\sum_{n=1}^{\infty} \frac{1}{2n^2-3}$, c) $\sum_{n=1}^{\infty} \frac{3n^2}{2n^2+2}$,
ć) $\sum_{n=1}^{\infty} \frac{n^2+3}{n^5-4n}$, d) $\sum_{n=3}^{\infty} \frac{2}{\sqrt{3n-6}}$, e) $\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{1+2+\cdots+n}}$, ę) $\sum_{n=1}^{\infty} \frac{\pi}{\sqrt{3n^3-n}}$,
f) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+\sqrt{n}}$, g) $\sum_{n=1}^{\infty} \frac{e+\cos(\pi^n)}{\sqrt{n}}$, h) $\sum_{n=1}^{\infty} \sin\frac{1}{\sqrt{n}}$, i) $\sum_{n=1}^{\infty} \sin\frac{1}{n^2}$,
j) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin\frac{1}{\sqrt{n}}$, k) $\sum_{n=1}^{\infty} \cos\frac{1}{n^2}$, l) $\sum_{n=1}^{\infty} \frac{1+2+\cdots+n}{\sqrt{n^3}} \sin\frac{1}{n+1}$, ł) $\sum_{n=1}^{\infty} \frac{1}{n}\cos\frac{1}{n}$,
m) $\sum_{n=1}^{\infty} \frac{1}{e^n+\pi^n}$, n) $\sum_{n=1}^{\infty} \frac{n!}{e^n}$, ń) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$, o) $\sum_{n=2}^{\infty} n\cdot\sin\frac{\pi}{2^n}$,
ó) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$, p) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$, r) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}$, s) $\sum_{n=1}^{\infty} \frac{n^n}{3^n\cdot(n!)^2}$,
ś) $\sum_{n=1}^{\infty} \frac{3^n-2^n}{5^n-4^n}$, t) $\sum_{n=1}^{\infty} (\frac{2008}{n})^n$, u) $\sum_{n=1}^{\infty} \frac{1}{n^2}\cdot(\frac{3}{5})^n$, w) $\sum_{n=1}^{\infty} (\frac{n}{n+1})^{n^2}$,
x) $\sum_{n=1}^{\infty} \frac{1}{\pi^n}(\frac{n+1}{n})^{n^2}$, y) $\sum_{n=1}^{\infty} \frac{n^2}{(3+\frac{7}{n})^n}$, z) $\sum_{n=1}^{\infty} (\arctan\frac{1}{n})^n$, ż) $\sum_{n=1}^{\infty} (\operatorname{arccot}\frac{1}{n})^n$,
ź) $\sum_{n=1}^{\infty} (\frac{n-1}{n+1})^n$.

Exercise. 4 Check the convergence of the following alternating series (absolutely convergent, conditionally convergent or divergent).

a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^3+2n^2+6}}$, b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+3}}$, c) $\sum_{n=1}^{\infty} (-1)^n\frac{n-3}{n^3+2n}$,
d) $\sum_{n=1}^{\infty} (-1)^n\frac{n^2+6}{7-2n^2}$, e) $\sum_{n=1}^{\infty} (-1)^n\pi^{-n}$, f) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$,
g) $\sum_{n=1}^{\infty} (-1)^n\frac{e^n}{n^n}$, h) $\sum_{n=1}^{\infty} (-1)^n(\sqrt{n+1}-\sqrt{n})$, i) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$.

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