

Exercise. 1 Calculate the following double integrals bounded by region D .

- a) $\int_D \int (x \ln y + xy^2) dx dy$, $D = [0, 2] \times [1, e]$ *answer:* $\frac{2e^3}{3} + \frac{4}{3}$
- b) $\int_D \int \frac{\cos y}{1+x^2} dx dy$, $D = [0, 1] \times [0, \frac{\pi}{2}]$ *answer:* $\frac{\pi}{4}$
- c) $\int_D \int (y-x) dx dy$, $D : y = x^2 - 2x, y = x$ *answer:* -4.05
- d) $\int_D \int 2xy dx dy$, $D : y = x, y = \frac{1}{x}, x = e$ *answer:* $\frac{e^4 - 5}{4}$
- e) $\int_D \int \frac{2x}{y} dx dy$, $D : y = \sqrt{x}, y = \frac{1}{x}, y = \frac{1}{2}$ *answer:* $\frac{81}{64}$
- f) $\int_D \int \frac{x}{\sqrt{x^2 + y^2}} dx dy$, $D : x^2 + y^2 \leq 4, x^2 + y^2 \geq 1, y \geq 0, y \geq x$ *answer:* $-\frac{3\sqrt{2}}{4}$
- g) $\int_D \int xy^2 dx dy$, $D : x^2 + y^2 \leq 4, x \geq 0$ *answer:* $\frac{64}{15}$

Exercise. 2 Find the volume of the solids bounded by surfaces.

- a) $x^2 + y^2 = 1, x + y + z = 3, z = 0$ *answer:* 3π
- b) $z = \sqrt{x^2 + y^2}, x^2 + y^2 + z^2 = 8$ *answer:* $\frac{32\pi}{3}(\sqrt{2} - 1)$
- c) $4x + 2y + z = 8, x \geq 0, y \geq 0, z \geq 0$ *answer:* $\frac{32}{3}$

Exercise. 3 Calculate the following triple integrals bounded by solid V .

- a) $\int_V \int \int \frac{dx dy dz}{yz}$, $V = [1, 2] \times [1, e] \times [1, e]$ *answer:* $\frac{3}{2}$
- b) $\int_V \int \int z \sqrt{x^2 + y^2} dx dy dz$, $V : x^2 + y^2 = 16, x^2 + y^2 = 9, z = 2, z = 4$ *answer:* 148π
- c) $\int_V \int \int \frac{dx dy dz}{\sqrt{x^2 + y^2 + z^2}}$, $V : 4 \leq x^2 + y^2 + z^2 \leq 9$ *answer:* 10π

Author: mgr Mariusz Kaszubowski.