

Ex. 1 a) (2 pts) Write the definition of the sum of the series $\sum_{n=1}^{\infty} a_n$.

b) Establish whether the following series converge or diverge:

$$\text{a) } \sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right), \text{ (2 pts)} \quad \text{b) } \sum_{n=1}^{\infty} \frac{2^n n!}{n^2}, \text{ (3 pts)} \quad \text{c) } \sum_{n=1}^{\infty} \frac{n^2}{5^n}, \text{ (3 pts)}$$

Ex. 2 a) (3 pts) Calculate the limit $\lim_{(x,y) \rightarrow (1,1)} \frac{x^4 - y^4}{x^3 - y^3}$.

b) (5 pts) Find all second partial derivatives of the function $f(x, y) = \sin(x - \ln y)$.

c) (6 pts) Calculate the approximated value of the number $\frac{(1.2) \cdot (2.3)}{(1.2)^2 + (2.3)^2}$.

Ex. 3 (8 pts) Find all local extremes of the function $f(x, y) = 2xy - 3x^2 - 2y^2 + 10$.

Ex. 4 a) (2 pts) Give two applications of a double integral (formulas and graphs – if necessary).

b) (6 pts) Calculate $\iint_D y dx dy$ if $D = \{(x, y) : 1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0, y \leq x\}$.

Ex. 5 (2×5 pts) Solve the following equations:

$$\text{a) } y' = \frac{y-x}{x} \text{ (for } x \neq 0), \quad \text{b) } x^2 y' + y = x \text{ (for } x > 0) \text{ – use the integrating factor.}$$