FINAL EXAM IN MATHEMATICS FOR EPM III - JANUARY 25TH

Ex. 1 *(3×3 pts)* Establish whether the series is convergent or divergent:

$$\text{a) } \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}, \qquad \text{b) } \sum_{n=1}^{\infty} \sin\left(\frac{\pi n+1}{2n+1}\right), \qquad \text{c) } \sum_{n=1}^{\infty} \frac{n^2}{6^n}.$$

Ex. 2 *(2+4 pts)* a) Write the definition of the sum of the series $\sum_{n=1}^{\infty} a_n$.

b) Use the above definition to find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

Ex. 3 *(4 pts)* Show that the limit $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ doesn't exist. Use any method you like.

Ex. 4 *(5 pts)* Show that the function $f(x,y) = x\sin y + y\sin x$ is a solution of the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f(x,y)$.

Ex. 5 *(8 pts)* Find all extremes and saddle points of the function $f(x,y) = 4xy - x^4 - y^4$.

Ex. 6 *(6 pts)* Calculate $\iint_D \frac{x}{x^2+y^2}dxdy$ if $D = \{(x,y) : x^2+y^2 \le 16,\ y \ge x\}$. Use polar coordinates.

Ex. 7 *(2×2 pts)* a) Describe spherical coordinates in the Cartesian space $\mathbf{R}^3$.

b) Give two applications of a double integral (formulas and graphs – if necessary).

Ex. 8 *(2×4 pts)* Solve the following equations:

$$\text{a) } y' = -y^2 e^x, \qquad \text{b) } y' - \frac{1}{x}y = x^2 \text{ for } x > 0 \text{ (use the integrating factor).}$$