Exercise 1 a) Give the necessary condition for convergence of a series $\sum\limits_{n=1}^{\infty} a_n$.

b) Check, whether the following series converge or diverge: $\sum\limits_{n=1}^{\infty} \frac{n+2}{n+4}$, $\sum\limits_{n=1}^{\infty} \frac{4^n}{n!}$, $\sum\limits_{n=1}^{\infty} \frac{n}{n^3+1}$.

Exercise 2 a) Compute the limit $\lim\limits_{(x,y)\to(0,2)} \frac{\sin(xy)}{x}$.

b) Compute the second-order partial derivatives of the function $f(x,y) = 4x + y \cdot \sin x - y^4$.

c) Find the approximated value of $\ln(\frac{2.01}{1.98})$ using the total differential.

Exercise 3. Find the local extreme values of the function $f(x,y) = 4x - x^4 - y^4$.

Exercise 4 a) Compute the double integral $\iint\limits_D x \cdot y dxdy$ over the region D, where D is bounded by the lines $y = x$, $\quad y = 2x$, $\quad x = 1$. Draw the region of integration.

b) Describe the procedure of changing the Cartesian integral $\iint\limits_R f(x,y)dxdy$ into a polar integral and compute $\iint\limits_R \sqrt{x^2+y^2}dxdy$, where R is the part of the ring between circles $x^2+y^2=1$ and $x^2+y^2=4$ in the second quadrant.

Exercise 5. Solve the following differential equations:

a) $y' = \frac{x+y}{x}$,

b) $y' - y = 5e^{2x}$,

c) $y'' - y = e^x + x^2$.