Exercise 1 a) Give the definition of the sum of the series $\sum_{n=1}^{\infty} a_n$.

b) Using the definition of the sum and the partial fractions find the sum of the following series: $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$.

c) Check, whether the following series converge or diverge: $\sum_{n=1}^{\infty} (1-\frac{1}{n})^n$, $\sum_{n=1}^{\infty} \frac{(n+3)!}{n!3^n}$.

Exercise 2 a) Calculate the limit $\lim_{(x,y)\to(1,1)} \frac{x^3-y^3}{x-y}$.

b) Find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial y \partial x}$ of function $f(x) = x^2 y + \cos y + y \sin x$.

Exercise 3. Find extreme values of the function $f(x,y) = e^y(x^2+y)$.

Exercise 4 a) Describe the procedure of changing the Cartesian integral $\iint_R f(x,y)dxdy$ into a polar integral.

b) Calculate $\iint_R e^{x^2+y^2} dxdy$, where R is a part of the ring between circles $x^2+y^2=1$ and $x^2+y^2=4$ contained in the first quadrant.

Exercise 5. Using the triple integral find the volume of a solid bounded from below by the plane $z=0$ and from above by the plane $x+y+z=4$ in the first octant ($x,y,z \geq 0$). Draw this solid.

Exercise 6. Solve the following differential equations:

a) $y' = (x+1)^2 y$,

b) $y' - 2y = 5e^{2x}$,

c) $y'' - 3y' = e^{3x} + x^2$.