GROUP A

Exercise 1. *(2 pts)* Using a double integral calculate the area of region $R = \{(x, y) : 0 \leq x \leq 2,\ 0 \leq y \leq 2x^2\}$. Draw the region as well.

Exercise 2. *(3 pts)* Using a double integral calculate the volume of a solid with base bounded by a curve $x^2 + y^2 = 1$, the base of this solid lies on the OXY plane. The solid is bounded from above by curve $2x^2 + 2y^2$.

Exercise 3. *(4 pts)* Using a triple integral calculate the mass of the solid given in the previous exercise, knowing that its density function is $f(x, y, z) = 2(x^2 + y^2 + z^2)$.

Exercise 4. *(1 pt)* Give a definition of cylindrical coordinates.

Exercise 5. *(2 pts)* Solve a differential equation $\frac{x}{4} \cdot \frac{dy}{dx} = \frac{1+y^2}{-y}$ using the method of separation of variables.

Exercise 6. *(2 pts)* Solve a homogenous differential equation $xy\frac{dy}{dx} = \frac{y^2-x^2}{4}$.

Exercise 7. *(4 pts)* Solve a linear differential equation $\frac{dy}{dx} - 3y = x^2 + 2$.

Exercise 8. *(2 pts)* Check that $\sin x + Ax + B$ is a solution to $y'' = -\sin x$ for any $A, B \in \mathbf{R}$.

GROUP B

Exercise 1. *(2 pts)* Using a double integral calculate the area of region $R = \{(x, y) : x^2 + y^2 \leq 1,\ y \geq x\}$. Draw the region as well.

Exercise 2. *(3 pts)* Using a double integral calculate the volume of a tetrahedron bounded by the plane $2x + 2y + z - 8 = 0$ and the axes of the coordinate system. Draw this tetrahedron.

Exercise 3. *(4 pts)* Using a triple integral calculate the mass of the solid given in the previous exercise, knowing that its density function is $f(x, y, z) = 2(x + y + z)$.

Exercise 4. *(1 pt)* Give a definition of spherical coordinates.

Exercise 5. *(2 pts)* Solve a differential equation $-x \cdot \frac{dy}{dx} = \frac{1+y^2}{2y}$ using the method of separation of variables.

Exercise 6. *(2 pts)* Solve a homogenous differential equation $2xy\frac{dy}{dx} = y^2 - x^2$.

Exercise 7. *(4 pts)* Solve a linear differential equation $x\frac{dy}{dx} - y = x^2$.

Exercise 8. *(2 pts)* Check that $A\sin x + B\cos x$ is a solution to $y'' + y = 0$ for every $A, B \in \mathbf{R}$.

GROUP C

Exercise 1. *(2 pts)* Using a double integral calculate the area of region $R = \{(x, y) : x^2 \leq y \leq 18 - x^2\}$. Draw the region as well.

Exercise 2. *(3 pts)* Using a double integral calculate the volume of a solid with base bounded by a curve $x^2 + y^2 = 4$, the base of this solid lies on the OXY plane. The solid is bounded from above by a curve $\sqrt{5x^2 + 5y^2}$.

Exercise 3. *(4 pts)* Using a triple integral calculate the mass of the solid given in the previous exercise, knowing that its density function is $f(x, y, z) = x^2 + y^2 + z^2$.

Exercise 4. *(1 pt)* Give a definition of polar coordinates. Draw an example of changing Cartesian (OXY) coordinates to polar coordinates.

Exercise 5. *(2 pts)* Solve a differential equation $xy \cdot \frac{dy}{dx} = \frac{1-3y^2}{4}$ using the method of separation of variables.

Exercise 6. *(2 pts)* Solve a homogenous differential equation $4xy\frac{dy}{dx} = x^2 + y^2$.

Exercise 7. *(4 pts)* Solve a linear differential equation $x\frac{dy}{dx} - 2y = x^2$.

Exercise 8. *(2 pts)* Check if e^{4x} is a solution to $y''' - y'' + y'' + y = 0$.