

**Points in each group:****Exercise 1 a.** (1 pt)**Exercise 1 b.** (2 pts)**Exercise 2 a.** (1 pt)**Exercise 2 b.** (2 pts)**Exercise 3.** (3 pts)**Exercise 4.** (4 pts)**Exercise 5.** (3 pts)**Exercise 6.** (3 pts)**Exercise 7.** (1 pt)**GROUP A****Exercise 1 a.** Give a definition of the ratio test for convergence of a series.**Exercise 1 b.** Use the ratio test to establish convergence of  $\sum_{n=1}^{\infty} 2 \cdot \frac{7^n}{n!}$ .**Exercise 2 a.** Give the definition of absolute convergence of a series.**Exercise 2 b.** Establish, if the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$  is absolutely convergent.**Exercise 3.** Find and draw the domain of function  $f(x, y) = \frac{1}{\sqrt{1-x^2-y^2}}$ .**Exercise 4.** Find the extremes of function  $f(x, y) = (x - y + 2)^2 + (2x + y - 6)^2$ .**Exercise 5.** Calculate the approximated value of  $0.99 \cdot \ln(1.02)$ .**Exercise 6.** Let  $f(x, y) = x^2 + y^2 + xy^2 + yx^2$ . Check, if it is true (or false) that  $\frac{\partial f}{\partial x}(x, y) - \frac{\partial f}{\partial y}(x, y) + 2(y - x) = y^2 - x^2$ .**Exercise 7.** Draw the graph of an ellipsoid (mark all axes and all radiuses) and write down its formula.**GROUP B****Exercise 1 a.** Give a definition of the root test for convergence of a series.**Exercise 1 b.** Use the root test to establish convergence of  $\sum_{n=1}^{\infty} \left(\frac{\pi}{n^2}\right)^n$ .**Exercise 2 a.** Give the definition of conditional convergence of a series.**Exercise 2 b.** Establish, if the series  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{(n-1)^4}$  is absolutely convergent.**Exercise 3.** Find and draw the domain of function  $f(x, y) = \ln(x^2 + y - 2)$ .**Exercise 4.** Find the extremes of function  $f(x, y) = 2(x + 1)^2 + 3(y + 2)^2$ .**Exercise 5.** Calculate the approximated value of  $(1.06)^2 + 1.06 \cdot 1.97 + (1.97)^2$ .**Exercise 6.** Let  $f(x, y) = x^2 + y^2 + xy$ . Check, if it is true (or false) that  $\frac{1}{3} \left( \frac{\partial f}{\partial x}(x, y) + \frac{\partial f}{\partial y}(x, y) \right) = y + x$ .**Exercise 7.** Draw the graph of a parabolic cylinder (mark all the axes) and write down its formula.

## GROUP C

**Exercise 1 a.** How is the sequence of partial sums of a series connected with the convergence of that series?

**Exercise 1 b.** Use the limit of a sequence of partial sums to establish if the series  $\sum_{n=9}^{\infty} \frac{1}{(n-7)(n-8)}$  is convergent.

**Exercise 2 a.** Give the definition of a p-series and write the conditions under which it is convergent.

**Exercise 2 b.** Establish, if the series  $\sum_{n=12}^{\infty} (n-11)^{-3}$  is convergent.

**Exercise 3.** Show that the limit  $\lim_{(x,y) \rightarrow (1,1)} \frac{x+y-1}{x^2+y^2-1}$  does not exist using any method you want.

**Exercise 4.** Find the extremes of function  $f(x, y) = x^2y - 8x - 9y$ .

**Exercise 5.** Find the plane tangent to  $f(x, y) = x^{y+1}$  at a point  $P(3, 2, ?)$ .

**Exercise 6.** Let  $f(x, y) = x \sin y + y \sin x$ . Check, if it is true (or false) that  $\frac{\partial^2 f}{\partial x^2}(x, y) + \frac{\partial^2 f}{\partial y^2}(x, y) = f(x, y)$ .

**Exercise 7.** Draw the graph of a hyperboloid of one sheet (mark all axes) and write down its formula.

## GROUP D

**Exercise 1 a.** Give the definition of an integral test for convergence of a series.

**Exercise 1 b.** Use the integral test to establish if the series  $\sum_{n=4}^{\infty} \frac{\sqrt{n}}{n+1}$  is convergent.

**Exercise 2 a.** Give the definition of a necessary condition for convergence of a series.

**Exercise 2 b.** Show that the series  $\sum_{n=1}^{\infty} \sqrt[n]{\frac{n}{100}}$  is divergent.

**Exercise 3.** Show that the limit  $\lim_{(x,y) \rightarrow (\pi,0)} \frac{x^2y}{x^4+y^2-1}$  does not exist using any method you want.

**Exercise 4.** Find the extremes of function  $f(x, y) = 15x + 8y - xy^2$ .

**Exercise 5.** Find the plane tangent to  $f(x, y) = (y+1)^x$  at a point  $P(2, 3, ?)$ .

**Exercise 6.** Let  $f(x, y) = \ln x \ln y$ . Check, if it is true (or false) that  $\frac{\partial^2 f}{\partial x^2}(x, y) \cdot \frac{\partial^2 f}{\partial y^2}(x, y) = f(x, y) \cdot x^2y^2$ .

**Exercise 7.** Draw the graph of a hyperboloid of two sheets (mark all axes) and write down its formula.