

Ex. 1 Check (using the definition) the convergence and find the sum of the following series:

a) $\sum_{n=1}^{\infty} e^{1-n},$

b) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right),$

c) $\sum_{n=1}^{\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}}.$

Ex. 2 Check the type of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3 + 2n^2 + 6}}.$

Ex. 3 Find the limit $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - y^2}{x - y}$ where $x \neq y.$

Ex. 4 Find the partial derivatives $\frac{\partial f}{\partial x}(1, 1)$ and $\frac{\partial f}{\partial y}(1, 1)$ if $f(x, y) = \frac{1}{x + y}.$

Ex. 5 Give the summary of max-min tests. Test the function $f(x, y) = x^2 + y^2 - 2x + 4y + 6$ for minima, maxima and saddle points.

Ex. 6 Find the absolute maxima and minima of the function $f(x, y) = x^2 + xy + y^2 - 6x$ on the rectangle plate $0 \leq x \leq 5, -3 \leq y \leq 3.$

Ex. 7 By considering different lines of approach show that the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.