Ex. 1 Prove or disprove the convergence of series:

1. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$,
2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{1}{\sqrt{n}}$,

Ex. 2 Check the type of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2}$.

Ex. 3 Find the extreme values of the function $f\colon \mathbf{R}^2 \to \mathbf{R}$: $f(x,y) = 2x^2 + y^2 + 4x - 4y + 5$.

Ex. 4 Find the absolute minimum and absolute maximum of $f : \mathbf{R}^2 \to \mathbf{R}$, $f(x,y) = x + y$, on the plane bounded by the lines: $y = 1$, $y = x^2$.

Ex. 5 Show that:

1. $u_x + u_y = 1$, if $u = \ln(e^x + e^y)$,
2. $u_{xy} = u_{yx}$, if $u = x \sin y + y \sin x$.

Ex. 6 Write the *necessary* condition for the existence of an extremal point, given a differentiable function of two variables. Give an example of such function, which doesn't fulfill this necessary condition *in any point*.

Ex. 7 Give an example of a divergent series, which fulfills the necessary condition of convergence.