

Final exam after the 2nd semester – June 18th 2009

Ex. 1 Compute the following integrals: (a) $\int x^2 \cos x dx$, (b) $\int \frac{\ln^2 x}{x} dx$, (c) $\int \frac{dx}{x^2+2x}$.

Ex. 2

- (a) Give two examples of applications of definite integrals, draw diagrams (if needed).
(b) Find the area of a region between two curves: $y = x^2$ and $x = y^2$.

Ex. 3 Find $\sqrt[3]{i}$ and indicate their placement in the complex plane.

Ex. 4

- (a) Solve the system of linear equations using the method of Gaussian elimination:
$$\begin{cases} x + y & = & 1, \\ x + 2y - 3z & = & -3, \\ 2x + 4y + z & = & 1. \end{cases}$$
- (b) Give three properties of a determinant.

(c) Compute $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & -1 & 1 \end{vmatrix}$. Do not use the Sarrus' method!

Ex. 5 Let $A = \begin{bmatrix} 1 & 3 \\ 0 & -4 \end{bmatrix}$.

- (a) Find the eigenvalues of A and eigenvectors associated with them.
(b) Find the eigenvalues of A^{-1} , $3A$, A^3 and $A - 5I$.

Ex. 6

- (a) The area of a parallelogram spanned by vectors \vec{p} and \vec{q} is 10. Compute the area of a triangle spanned by vectors $2\vec{p} + \vec{q}$ and $3\vec{p} - 2\vec{q}$.
(b) Find the general equation of a plane π that passes through the following points: $A = (1, 0, 1)$, $B = (3, 1, 0)$ and $C = (0, 0, -2)$.