Final exam after the 2nd semester – September 23rd 2009

Ex. 1 Compute the following integrals: (a) $\int x^2 \sin x dx$, (b) $\int \frac{\arctan^3 x}{1+x^2} dx$, (c) $\int \frac{dx}{x^2-4}$.

Ex. 2

(a) Give two examples of applications of definite integrals, draw diagrams (if needed).

(b) Find the area of a region between two curves: $y = x^2$ and $y = \sqrt{x}$.

Ex. 3 Compute $\frac{(1+i)^2}{1-i}$ and write the answer in algebraic form.

Ex. 4

(a) Solve the system of linear equations using the method of Gaussian elimination:
$$\begin{cases} x + 2y + 3z &= 14, \\ 3x + y + 2z &= 11, \\ 2x + 3y + z &= 11. \end{cases}$$

(b) Give three properties of a determinant of a matrix.

(c) Compute the determinant of the following matrix $A = \begin{bmatrix} 0 & 5 & 0 & 2 \\ 8 & 3 & 4 & 5 \\ 7 & 2 & 1 & 4 \\ 0 & 4 & 0 & 0 \end{bmatrix}$. Do not use the Sarrus' method!

Ex. 5 Let $A = \begin{bmatrix} 2 & 17 \\ 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A and eigenvectors associated with them.

(b) Find the eigenvalues of $A^{-1}, 5A, A^3$ and $A - 7I$.

Ex. 6

(a) The area of a parallelogram spanned by vectors $\vec{p}$ and $\vec{q}$ is 10. Compute the area of a triangle spanned by vectors $2\vec{p} - \vec{q}$ and $3\vec{p} + 2\vec{q}$.

(b) Find the general equation of a plane π that passes through the following points: $A = (-1, 0, 0)$, $B = (3, 1, 0)$ and $C = (0, 0, 2)$.