Final exam after the 2nd semester – September 10th 2009

Ex. 1 Compute the following integrals: (a) $\int x^2 e^x dx$, (b) $\int \frac{\ln^{10} x}{x} dx$, (c) $\int \frac{dx}{x^2-1}$.

Ex. 2

(a) Give two examples of applications of definite integrals, draw diagrams (if needed).

(b) Find the area of a region between two curves: $y = x^3$ and $x = y^2$.

Ex. 3 Compute $(\frac{1+i}{2})^{18}$ using the de Moivre's formula and write the answer in algebraic form.

Ex. 4

(a) Solve the system of linear equations using the method of Gaussian elimination:
$$\begin{cases} x + y & = & 1, \\ x + 2y - 3z & = & -3, \\ 2x + 4y + z & = & 1. \end{cases}$$

(b) Give three properties of a rank of a matrix.

(c) Compute the rank of the following matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & -1 & 1 \end{bmatrix}$.

Ex. 5 Let $A = \begin{bmatrix} 1 & 0 \\ 3 & -4 \end{bmatrix}$.

(a) Find the eigenvalues of A and eigenvectors associated with them.

(b) Find the eigenvalues of $A^{-1}, 3A, A^3$ and $A - 5I$.

Ex. 6

(a) Find the line including the point $A = (1, -2, 3)$ and perpendicular to the plane $\pi : 2x + 4y - z + 1 = 0$.

(b) Compute $|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|$ knowing that $|\vec{a} \times \vec{b}| = 5$.