

**Group A.****Ex. 1**

- a) Write the definition of the dot product. (1 point)
- b) For which value of  $k$  is vector  $\vec{a} = [1, 2, 3]$  perpendicular to vector  $\vec{b} = [0, k, 2]$ ? (2 points)

**Ex. 2** Solve matrix equation (find  $X$ )  $3X - AX = B$  if  $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & -3 \\ 2 & -1 \end{bmatrix}$ . (3 points)

**Ex. 3** Solve the system of linear equations using the method of Gaussian elimination.

$$\begin{cases} x + 2z & = & -6 \\ 3x + y + z & = & -1 \\ 3x + 2z & = & 0 \end{cases} \quad (3 \text{ points})$$

**Ex. 4** Find the unknown  $x$  using the Cramer's rule (do not use the Sarrus' method at any time):

$$\begin{cases} 3x + 7y + 2z + 4t & = & 0 \\ 2y + z & = & 0 \\ x + 4y + z & = & 1 \\ 5x + 3y + 2z & = & 0 \end{cases} \quad (3 \text{ points})$$

**Ex. 5** Let  $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$ . Find the eigenvalues of  $A$ ,  $A^{-1}$ ,  $5A$ ,  $A^2$ ,  $A - 6I$ . (3 points)

**Group B.****Ex. 1**

- a) Write the definition of the dot product. (1 point)
- b) For which value of  $k$  is vector  $\vec{a} = [1, k, 2]$  perpendicular to vector  $\vec{b} = [2, 7, 13]$ ? (2 points)

**Ex. 2** Solve matrix equation (find  $X$ )  $AX - 2X = B$  if  $A = \begin{bmatrix} -1 & -5 \\ 2 & -3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 & -2 \\ 5 & 1 & 7 \end{bmatrix}$ . (3 points)

**Ex. 3** Solve the system of linear equations using the method of Gaussian elimination.

$$\begin{cases} x + y & = & 1 \\ x + 2y - 3z & = & -3 \\ 2x + 4y + z & = & 1 \end{cases} \quad (3 \text{ points})$$

**Ex. 4** Find the unknown  $x$  using the Cramer's rule (do not use the Sarrus' method at any time):

$$\begin{cases} 3x + 7y + 2z + 4t & = & 0 \\ 2y + z & = & 0 \\ x + 4y + z & = & 1 \\ 5x + 3y + 2z & = & 0 \end{cases} \quad (3 \text{ points})$$

**Ex. 5** Let  $A = \begin{bmatrix} 1 & 3 \\ 0 & -4 \end{bmatrix}$ . Find the eigenvalues of  $A$ ,  $A^{-1}$ ,  $3A$ ,  $A^3$ ,  $A - 5I$ . (3 points)