FINAL (REPEAT) EXAM IN MATHEMATICS FOR EPM II - SEPTEMBER 19TH, 2012

Ex. 1 *(4+4+4 pts)* Calculate the following integrals:

$$\text{a) } \int \left(\frac{3\sin(\ln x)}{x} + 5\sin x e^{\cos x}\right)dx, \quad \text{b) } \int (x^2-1)\sin x dx, \quad \text{c) } \int \frac{2x+3}{x^2-x}dx.$$

Ex. 2 *(2+4 pts)* a) Give two applications of definite integrals different from the one used in **Ex. 2b** – write formulas and draw diagrams.

b) Find the volume of the solid of revolution created by rotating the curve $y = x\sqrt{\sin x}$ around the OX-axis for $x \in [0, \frac{\pi}{2}]$.

Ex. 3 *(4 pts)* Under which values of k is the matrix $A = \begin{bmatrix} 4k & 5k \\ 1 & 2k \end{bmatrix} + \begin{bmatrix} -2k & 1 \\ 0 & k \end{bmatrix}$ singular?

Ex. 4 *(5 pts)* Establish the number of solutions of the following system (without solving it!)

$$\begin{cases} 3x + y - 2z + t = 1 \\ y + 3z - 3t = 0 \\ 2x + 2y + z - t = 0 \end{cases}$$

Ex. 5 *(4+3 pts)* Let $A = \begin{bmatrix} 2 & 5 \\ 0 & -2 \end{bmatrix}$. a) Find the eigenvalues and eigenvectors of A.

b) Find the spectrum of $(A^{-1} + 3I)^3$.

Ex. 6 *(4 pts)* Compute the area of the triangle with vertices: $A = (3, 1, 0)$, $B = (0, 1, 0)$ and $C = (1, 0, -4)$.

Ex. 7 *(5 pts)* Find the general equation of the plane π that passes through points: $A = (3, 0, 1)$, $B = (3, 1, 0)$ and $C = (0, 0, 3)$.

Ex. 8 *(3+4 pts)* a) Calculate $Re(\frac{1}{z}) + \frac{2|z|}{Im(z)}$, if $z = 5 + 4i$. b) Solve the equation $z \cdot \overline{z} + 3z + 5(i+1)^2 = 0$.