FINAL EXAM IN MATHEMATICS FOR EPM II - JUNE 1ST

Ex. 1 *(3+4+4 pts)* Calculate the following integrals: a) $\int \frac{3\cos(\ln x)+\sin(\ln x)}{x}dx$, b) $\int (x^2+2x)\cos x dx$, c) $\int \frac{3x+2}{x^3+x}dx$.

Ex. 2 *(2+4 pts)* a) Give two applications of definite integrals different from the one used in **Ex. 2b** – write formulas and draw diagrams.

b) Find the volume of the solid of revolution created by rotating the curve $y = x\sqrt{\ln x}$ around the OX-axis for $x \in [1, e]$,

Ex. 3 *(2+3 pts)* a) Calculate $Im(\frac{1}{z}) + \frac{2|z|}{Re(z)}$, if $z = 4 - 3i$. b) Solve the equation $z \cdot \overline{z} + 2z - 2i = 0$.

Ex. 4 *(4 pts)* Under what values of k is the matrix $A = \begin{bmatrix} 4k & 5k \\ 1 & 2k \end{bmatrix} - \begin{bmatrix} -2k & 1 \\ 0 & k \end{bmatrix}$ invertible?

Ex. 5 *(4 pts)* Establish the number of solutions of the following system (without solving it!)

$$\begin{cases} 2x + y - z + t = 1 \\ y + 3z - 3t = 1 \\ x + y + z - t = 1 \end{cases}$$

Ex. 6 *(4+3 pts)* Let $A = \begin{bmatrix} 1 & 7 \\ 0 & 3 \end{bmatrix}$. a) Find the eigenvalues and eigenvectors of A.

b) Find the spectrum of $(A^{-1} - 3I)^4$.

Ex. 7 *(4+4 pts)* a) Compute the volume of the tetrahedron V with vertices: $A = (2, 2, 0)$, $B = (-1, 2, 2)$, $C = (-1, 1, 4)$ and $D = (0, 0, 1)$.

b) Give the definition and at least three properties of the cross product.

Ex. 8 *(5 pts)* Find the general equation of the plane π that passes through points: $A = (1, 0, 3)$, $B = (3, 1, 0)$ and $C = (0, 0, 4)$.