

**(Ex 1)** (1.5 pts) Calculate the following limits.

a)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^4 - 1} =$

b)  $\lim_{x \rightarrow 0} \frac{\sin(3x) + 2x}{\sin(5x) + 3x} =$

c)  $\lim_{x \rightarrow \infty} \sin(\arctan(x)) =$

**(Ex 2)** (2 pts) Discuss continuity of the following function.

$$f(x) = \begin{cases} 6 & x < -3 \\ \left(\frac{1}{3}\right)^x + 2 & -3 \leq x \leq 0 \\ \frac{e^x}{x} & x > 0 \end{cases}$$

**(Ex 2)** (2 pts) Calculate derivatives.

a)  $(5\sqrt[4]{x} - 3x^2 + \frac{3}{5}\sqrt[5]{x^6} + 2^3)' =$

b)  $(\ln(5x) + \ln(x^2) + \tan x \cdot \cos x)'$

c)  $(3 \cdot 2^{2x} - 10^x e^x + \ln^2(x))' =$

d)  $(\arccos(\sqrt{x}) + e^{\pi^x})' =$

**(Ex 4)** (1.5 pts) Calculate multiple derivatives.

a)  $(xe^x)'' =$

b)  $(x^{91} + e^x)^{(106)} =$

c)  $(\sqrt{x} - \sin(x))''' =$

**(Ex 5)** (0.5 pt) Find the missing function.

$(?)' = \frac{2}{1+x^2} + \sin(x) + e^x + x$

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**(Ex 6)** (1.5 pts) Calculate the approximated value of  $\frac{1}{2.02}$  if you know that the exact value is 0.4950495. Also, calculate the absolute and the relative error.

$f(x) =$                        $f'(x) =$                        $x_0 =$                        $\Delta x =$

$\frac{1}{2.02} \approx$

Absolute error:  $E_A =$

Relative error:  $E_R =$

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**(Ex 7)** (2 pts) Find the largest and the smallest value of the function  $f(x) = \frac{2}{x} - 2 + x$  for  $x \in [1, 4]$ . Also, discuss its monotonicity in the given interval.

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**(Ex 8)** (2 pts) Find inflection points and intervals of concavity/convexity for  $f(x) = -x^4 + 12x^3 - 48x^2 + 60x + 1$ .

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**(Theory)** (1+0.5+0.5 pts) a) Give an example of an increasing convex function - both a graph and a formula.   
b) Give one application of derivatives that is not mentioned in this test.  
c) Give an example of a function  $f(x)$  such that the domain of  $f$  contains more points than the domain of  $f'$ .