

**Exercise. 1** Find the limits of the following sequences.

- a)  $\lim_{n \rightarrow \infty} \frac{(n+1)!}{(n+2)! - n!}$  *answer:* 0
- b)  $\lim_{n \rightarrow \infty} \frac{3^{n+1} - 4^n}{4^{n+1} - 2^n}$  *answer:*  $-\frac{1}{4}$
- c)  $\lim_{n \rightarrow \infty} (2n - \sqrt{4n^2 - 3n + 2})$  *answer:*  $\frac{3}{4}$
- d)  $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+2n}{(2-3n)(n+2)}$  *answer:*  $-\frac{2}{3}$
- e)  $\lim_{n \rightarrow \infty} \sqrt[n]{3^n + 4^n + |\sin n|}$  *answer:* 4
- f)  $\lim_{n \rightarrow \infty} \frac{2n^2 + \cos n}{n^2 + (-1)^n}$  *answer:* 2
- g)  $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n-1}\right)^{2n}$  *answer:*  $e^4$
- h)  $\lim_{n \rightarrow \infty} \left(\frac{2n-1}{2n+1}\right)^{n^2}$  *answer:* 0

**Exercise. 2** Find the limits of the following functions.

- a)  $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 + 3x + 2}$  *answer:* 4
- b)  $\lim_{x \rightarrow \infty} \arcsin(\sqrt{x^2 + x} - x)$  *answer:*  $\frac{\pi}{6}$
- c)  $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x^2 + 2x}$  *answer:* 1
- d)  $\lim_{x \rightarrow -\infty} (2 - x + 4x^2 - x^3)$  *answer:*  $\infty$
- e)  $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + 3x - 1} - \sqrt{x^2 - x})$  *answer:* -2
- f)  $\lim_{x \rightarrow \infty} \frac{6^{x+1} - 5^{x+1}}{3^{x+2} + 2 \cdot 6^{x-1}}$  *answer:* 18
- g)  $\lim_{x \rightarrow \infty} \left(\frac{3x+2}{3x-1}\right)^{2x}$  *answer:*  $e^2$
- h)  $\lim_{x \rightarrow \infty} \cos(\arctan(\frac{1-x}{1+x}))$  *answer:*  $\frac{\sqrt{2}}{2}$
- i)  $\lim_{x \rightarrow 2^+} e^{\frac{x-3}{x^2-x-2}}$  *answer:* 0
- j)  $\lim_{x \rightarrow 2^-} \arccos(\frac{\sin(x-2)}{|x-2|})$  *answer:*  $\pi$

**Exercise. 3** Check the continuity of the following function determining the type of discontinuity.

$$f(x) = \begin{cases} 3 + e^{\frac{2}{x}} & \text{for } x < 0 \\ 3 & \text{for } x = 0 \\ \frac{\sin(3x)}{x} & \text{for } 0 < x < 2 \\ x^2 - 3x & \text{for } 2 \leq x \leq 3 \\ \ln\left(\frac{1}{x-3}\right) & \text{for } x > 3 \end{cases}$$

*answer:* discontinuity of type I for  $x = 2$ , discontinuity of type II for  $x = 3$ .