

Exercise. 1 Set the natural domain of the following functions.

$$\begin{aligned}
 a(x) &= \sqrt{3x - x^2}, & b(x) &= \frac{1}{\sqrt[3]{x^6 - 1}}, & c(x) &= \sqrt[4]{x - \frac{1}{x}}, \\
 d(x) &= \sqrt{\frac{3}{x} - 2 - x}, & e(x) &= \sqrt[6]{\frac{2}{x} - x + 1}, & f(x) &= \sqrt{x - 4 + \frac{5}{x+2}}, \\
 g(x) &= \sqrt{\frac{x^2 + x - 6}{x^3 + 1}}, & h(x) &= \sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}, & i(x) &= \sqrt{2 - \frac{|x+1|}{|x-1|}}, \\
 j(x) &= \sqrt[4]{\sqrt{x-1} - 3 + x}, & k(x) &= \sqrt{1 - |x+1 - |x||}, & l(x) &= \sqrt{x^3 - 1} + \sqrt[4]{16 - x^4}, \\
 m(x) &= \frac{\sqrt{|x-4| - 2}}{\sqrt[3]{|x+2| - 8}}, & n(x) &= \frac{3x-6}{x^4 - 4x^2 - 5}, & o(x) &= \frac{\sqrt{x+1}}{(1-x)\sqrt{3-|x|}}, \\
 p(x) &= \frac{3}{\sqrt{||1-3x|-4||-6}}, & q(x) &= (|x^2 - 4| - 3x)^{-1}, & r(x) &= \frac{\sqrt[4]{5-|x-1|}}{|x|-5} + \frac{3x+4}{\sqrt{|x+1|-4}}, \\
 s(x) &= \frac{2x}{\sqrt{5-||3-2x|-1|}}, \\
 t(x) &= \frac{-x+2}{\sqrt{(x-2)(5-x)}} + \sqrt{x^2 - 7x + 12} - (|x| - 4)^{-1}.
 \end{aligned}$$

Exercise. 2 Find the range of the following functions.

$$\begin{aligned}
 a(x) &= \sqrt{x^2} + (\sqrt{x})^2 - x + 1, & b(x) &= \frac{x}{x-2}, & c(x) &= \frac{x^2-1}{|x-1|}, & d(x) &= \sqrt{4x - x^2}, \\
 e(x) &= \frac{x-1}{x^2-1}, & f(x) &= \frac{1}{2x+1}, & g(x) &= \frac{2}{\sqrt{1-x^2}}, & h(x) &= \sqrt[3]{-x^2 + 2x + 8}.
 \end{aligned}$$

Exercise. 3 Check if the functions f and g are equal.

$$\begin{aligned}
 \text{a) } f(x) &= \sqrt{(x-1)^2}, \quad g(x) = (\sqrt{x-1})^2, & \text{b) } f(x) &= 2 \cos^2 x, \quad g(x) = 1 + \cos 2x, \\
 \text{c) } f(x) &= \sqrt{x(x-1)}, \quad g(x) = \sqrt{x} \cdot \sqrt{x-1}, & \text{d) } f(x) &= \sin^2 x + \cos^2 x, \quad g(x) = \frac{\sqrt[3]{|x^3| + 3x^2 + |3x| + 1}}{\sqrt{x^2 - 1}}, \\
 \text{e) } f(x) &= \frac{x^2-9}{x-3}, \quad g(x) = x + 3.
 \end{aligned}$$

Exercise. 4 Check if the following functions are odd or even.

$$\begin{aligned}
 a(x) &= |x-3| + |x+3|, & b(x) &= x^2 \sqrt[3]{x-x^3}, & c(x) &= 2^x - 2^{-x}, \\
 d(x) &= \frac{4^x + 4^{-x}}{4^x - 4^{-x}}, & e(x) &= \sqrt[5]{(2-x)^2} + \sqrt[5]{x-2}, & f(x) &= x \cdot \frac{3^x - 1}{3^x + 1}, \\
 g(x) &= \sin |x| + |\sin x|, & h(x) &= \sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1+x}{1-x}}, & i(x) &= \log(x + \sqrt{1+x^2}).
 \end{aligned}$$

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