Exercise. 1 Sketch graphs of the following functions.

$a(x) = |x|,\quad b(x) = x|x|,\quad c(x) = \dfrac{\sqrt{x^2}}{x},$

$d(x) = \dfrac{1}{|x|},\quad e(x) = 2^{|x|},\quad f(x) = \dfrac{x^2-1}{|x-1|},$

$g(x) = \dfrac{|x+1|}{|x-1|},\quad h(x) = |2x+1|,\quad i(x) = |x+1|-2,$

$j(x) = 2|x-2|+1,\quad k(x) = \sqrt{x^2-4x+4},\quad l(x) = \sqrt{x^4-2x^2+1}-1,$

$m(x) = |x^2-x-6|,\quad n(x) = ||x-1|-2|,\quad o(x) = |x+1|\cdot(x-3),$

$p(x) = \dfrac{|x+1|+|x-1|}{2},\quad q(x) = |x+1|-|x+3|,\quad r(x) = |x-2|+|x+3|,$

$s(x) = \sin|x| + |\sin x|.$

Exercise. 2 Solve the following equations.

a) $|x+1| = 3,$ b) $||x|-1| = 4,$ c) $||x+1|-1| = 1,$

d) $|x+3| = |x-1|,$ e) $|x+3|-|5x-1| = 0,$ f) $|x-3|+|x-6| = 7,$

g) $|3-x|+|4+2x| = 5,$ h) $|x-3|+|x+1| = 6-x,$ i) $|2x+4|-|x-1|+|x-3| = 8.$

Exercise. 3 Solve the following inequalities.

a) $|x-4| \leq 2,$ b) $|x+3| > 4,$ c) $|x+2|+1 > x,$

d) $|x|+2x > 2,$ e) $|x|+|x-1| \geq 3,$ f) $|x+2|+|x| \geq 4,$

g) $|x|+|x-4| \leq 6-x,$ h) $|x+3|+2|x-1|-|2x-4| < 4,$ i) $\dfrac{|x+2|}{|x-2|} < 2,$

j) $|x+1-|x|| \leq 0,$ k) $|||x-1|-2|-3| > 1.$

Exercise. 4 Represent sets of points meeting the following conditions on the Cartesian plane.

a) $|x|+|y| = 1,$ b) $y+|x+2| \geq 2,$ c) $|x-y|+|x+y| = 2,$

d) $\begin{cases} |x+y| & = & 1 \\ y & = & |x| \end{cases},$ e) $\begin{cases} x^2-y^2 & = & 0 \\ |x|+|y| & < & 1 \end{cases},$ f) $|x|-|y| \geq 1.$

Definition: $|x| = \begin{cases} x & \text{for } x \leq 0 \\ -x & \text{for } x < 0 \end{cases}$

Properties:

a) $|x| \leq 0,$ b) $|x| = |-x|,$ c) $\sqrt{x^2} = |x|$, but $(\sqrt{x})^2 = x,$

d) $|x^n| = |x|^n,$ e) $|x\cdot y| = |x|\cdot|y|,$ f) $|\dfrac{x}{y}| = \dfrac{|x|}{|y|}$ for $y \neq 0,$

g) "triangle inequality": $|x+y| \leq |x|+|y|,$ h) $|x| = |y| \Leftrightarrow x = y \vee x = -y$

i) $|x| < a$ for $a \geq 0 \Leftrightarrow -a < x < a,$ j) $|x| > a$ for $a \geq 0 \Leftrightarrow x > a \vee x < -a.$

Author: mgr Mariusz Kaszubowski.