**Exercise 1.** Establish whether the series are convergent or divergent by finding the $n$th partial sum and checking its limit.

a) $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)}$, b) $\sum\limits_{n=1}^{\infty} \ln \frac{n}{n+1}$, c) $\sum\limits_{n=1}^{\infty} (-1)^{n+1}(2n-1)$, d) $\sum\limits_{n=1}^{\infty} \frac{4^n+2\cdot 5^n}{7^n}$,

e) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n 3^n}{6^n}$, f) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$, g) $\sum\limits_{n=1}^{\infty} \frac{1}{n^2+5n+6}$, h) $\sum\limits_{n=1}^{\infty} \sin{(2n-1)}\frac{\pi}{4}$,

i) $\sum\limits_{n=1}^{\infty} \frac{1}{(3n-2)(3n+1)}$, j) $\sum\limits_{n=2}^{\infty} \ln{(1-\frac{1}{n^2})}$, k) $\sum\limits_{n=1}^{\infty} \ln{(1+\frac{1}{n})}$, l) $\sum\limits_{n=1}^{\infty} \frac{7^n+3^n}{10^n}$,

m) $\sum\limits_{n=1}^{\infty} (\sqrt[n+1]{2} - \sqrt[n+2]{2})$, n) $\sum\limits_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}+n\sqrt{n+1}}$, o) $\sum\limits_{n=1}^{\infty} \frac{2+2^2+2^3+\cdots+2^n}{3^n}$, p) $\sum\limits_{n=0}^{\infty} \frac{3^{2n+1}}{2^{3n+2}}$.

*Answers: convergent – (a), (d), (e), (g), (i), (j), (l), (m), (n), (o), divergent – (b), (c), (f), (h), (k), (p).*

**Exercise 2.** Find the sums of the following geometric series.

a) $\sum\limits_{n=2}^{\infty} (-\frac{2}{3})^{n-1}$, b) $\sum\limits_{n=1}^{\infty} (-\frac{2}{3})^{n-1}$, c) $\sum\limits_{n=1}^{\infty} \frac{4}{5^n}$, d) $0.(9)$,

e) $0.(54)$, f) $\sum\limits_{n=1}^{\infty} (0.9998)^n$, g) $\sum\limits_{n=0}^{\infty} (\sqrt{17}-\sqrt{2}-\sqrt{3})^n$, h) $\sum\limits_{n=0}^{\infty} (\frac{1}{3^n} - \frac{1}{3^{n+1}})$.

*Answers: (a)* $-\frac{2}{5}$, *(b)* $\frac{3}{5}$, *(c)* 1, *(d)* 1, *(e)* $\frac{6}{11}$, *(f)* 4999, *(g)* $\frac{1}{1-(\sqrt{17}-\sqrt{2}-\sqrt{3})}$, *(h)* 1.

**Exercise 3.** Using the properties of harmonic/$p$-series establish, which of these series are convergent and which ones are not.

a) $\sum\limits_{n=3}^{\infty} \frac{1}{n}$, b) $\sum\limits_{n=1}^{\infty} n^{-\frac{2}{3}}$, c) $\sum\limits_{n=1}^{\infty} \frac{1}{n^4}$, d) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$,

e) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[\pi]{n^3}}$, f) $\sum\limits_{n=11}^{\infty} (n-10)^{-\frac{1}{2}}$, g) $\sum\limits_{n=1}^{\infty} \frac{n^\pi}{n^e}$, h) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^\pi}}$.

*Answers: convergent – (c), (h), divergent – (a), (b), (d), (e), (f), (g).*

**Nobody's perfect – if you notice any mistakes in answers, please let us know!**