Exercise 1. Check whether these series are convergent or divergent using the comparison test.

a) $\sum\limits_{n=1}^{\infty} \frac{1}{n^2+n}$, b) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$, c) $\sum\limits_{n=2}^{\infty} \frac{\ln n}{n^2}$, d) $\sum\limits_{n=2}^{\infty} \frac{n+1}{n^2-n}$,

e) $\sum\limits_{n=1}^{\infty} \frac{\arctan n}{n^2}$, f) $\sum\limits_{n=1}^{\infty} n \cdot \sin \frac{1}{n^3}$, g) $\sum\limits_{n=1}^{\infty} \frac{2+\sin n}{n}$, h) $\sum\limits_{n=1}^{\infty} \sqrt[n]{n} \cdot \tan \frac{1}{n}$.

Answers: convergent – (a), (c), (e), (f), divergent – (b), (d), (g), (h).

Exercise 2. Check whether these series are convergent or divergent using the simplified limit comparison test.

a) $\sum\limits_{n=1}^{\infty} \frac{2n^3-n^2+2}{n^5-n^3+3}$, b) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$, c) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$, d) $\sum\limits_{n=1}^{\infty} \frac{3^n-2^n}{4^n-3^n}$,

e) $\sum\limits_{n=1}^{\infty} (1-\cos\frac{1}{n})$, f) $\sum\limits_{n=1}^{\infty} \arcsin\frac{1}{n}$, g) $\sum\limits_{n=1}^{\infty} \frac{\arctan n}{n}$, h) $\sum\limits_{n=2}^{\infty} \frac{\ln n}{n^2}$.

Answers: convergent – (a), (d), (h), divergent – (b), (c), (e), (f), (g).

Exercise 3. Check whether these series are convergent or divergent using the ratio test.

a) $\sum\limits_{n=1}^{\infty} \frac{2^n}{n!}$, b) $\sum\limits_{n=1}^{\infty} \frac{2^n}{n^2}$, c) $\sum\limits_{n=1}^{\infty} \frac{(n!)^3}{(2n)!}$, d) $\sum\limits_{n=1}^{\infty} \frac{n^n}{n!}$,

e) $\sum\limits_{n=2}^{\infty} \frac{\ln n}{\pi^n}$, f) $\sum\limits_{n=1}^{\infty} \frac{3^n n!}{n^n}$, g) $\sum\limits_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$, h) $\sum\limits_{n=1}^{\infty} \frac{(2n)!(3n)!}{(5n)!}$.

Answers: convergent – (a), (e), (g), (h), divergent – (b), (c), (d), (f).

Exercise 4. Check whether these series are convergent or divergent using the root test.

a) $\sum\limits_{n=1}^{\infty} (\frac{n-1}{2n+1})^n$, b) $\sum\limits_{n=1}^{\infty} \frac{3^n+4^n}{2^n+5^n}$, c) $\sum\limits_{n=1}^{\infty} n \cdot (\frac{3}{5})^n$, d) $\sum\limits_{n=1}^{\infty} \frac{n^{100}}{\pi^n}$,

e) $\sum\limits_{n=1}^{\infty} (\frac{n+2}{n+3})^{n^2}$, f) $\sum\limits_{n=1}^{\infty} (\arccos\frac{1}{n})^n$, g) $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^n+1}$, h) $\sum\limits_{n=1}^{\infty} (\ln(2+\frac{1}{n}))^n$.

Answers: convergent – (a), (b), (c), (d), (e), (g), (h), divergent – (f).

Exercise 5. Check whether these series are convergent or divergent using the integral test.

a) $\sum\limits_{n=1}^{\infty} \frac{1}{3n+1}$, b) $\sum\limits_{n=1}^{\infty} \frac{1}{4n^2+9}$, c) $\sum\limits_{n=1}^{\infty} \frac{n}{e^{n^2}}$, d) $\sum\limits_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$,

e) $\sum\limits_{n=2}^{\infty} \frac{1}{n\ln n}$, f) $\sum\limits_{n=2}^{\infty} \frac{1}{n\cdot\ln n\cdot\ln(\ln n)}$, g) $\sum\limits_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$, h) $\sum\limits_{n=1}^{\infty} \frac{2^n}{16^n-1}$.

Answers: convergent – (b), (c), (d), (h), divergent – (a), (e), (f), (g).

Nobody's perfect – if you notice any mistakes in answers, please let us know!