

Exercise 1. Revise your knowledge of factorials — calculate values of the following expressions.

$$\text{a) } 2!, \quad \text{b) } 4!, \quad \text{c) } 2! \cdot 3!, \quad \text{d) } \binom{4}{1}, \quad \text{e) } \binom{3}{2}, \quad \text{f) } \frac{(n-1)!}{(n+1)!}, \quad \text{g) } \frac{(n+2)!(n+1)!}{n! \cdot (n-1)!}, \quad \text{h) } \frac{(n+1)!+n!}{(n+1)!-n!}, \quad \text{i) } \frac{\binom{n+2}{n}}{n^2}.$$

Answers: (a) 2, (b) 24, (c) 12, (d) 4, (e) 3, (f) $\frac{1}{n(n+1)}$, (g) $n(n+1)^2(n+2)$, (h) $\frac{n+2}{n}$, (i) $\frac{(n+1)(n+2)}{2n^2}$.

Exercise 2. Using the necessary condition for convergence of a series, justify the fact that these series are divergent.

$$\begin{aligned} \text{a) } \sum_{n=1}^{\infty} \frac{n+2}{n+100}, \quad & \text{b) } \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{100n^2+1}, \quad \text{c) } \sum_{n=2}^{\infty} \frac{n}{\ln n}, \quad \text{d) } \sum_{n=2}^{\infty} n \sqrt[n]{\frac{n}{100}}, \quad \text{e) } \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n, \\ \text{f) } \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n, \quad & \text{g) } \sum_{n=0}^{\infty} \frac{\pi^n}{e^n+3^n}, \quad \text{h) } \sum_{n=1}^{\infty} \frac{\arctan n}{\operatorname{arccot}(-n)}, \quad \text{i) } \sum_{n=1}^{\infty} \frac{(2n)!}{n^n}, \quad \text{j) } \sum_{n=1}^{\infty} \frac{(3n)!(5n)!}{(7n)!}. \end{aligned}$$

Nobody's perfect – if you notice any mistakes in answers, please let us know!