Exercise 1. Justify the convergence of the following series using the alternate series test.

a) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$, b) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{n+2}{n^2+3}$, c) $\sum\limits_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$,
d) $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{n}$, e) $\sum\limits_{n=1}^{\infty} (-1)^{n+1} (\frac{n+2}{3n-1})^n$, f) $\sum\limits_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$.

Exercise 2. Check whether these series are absolutely convergent.

a) $\sum\limits_{n=1}^{\infty} \frac{(-2)^n}{n^2}$, b) $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\frac{3}{2}}}$, c) $\sum\limits_{n=1}^{\infty} \frac{(1-2n)^n}{(3n+2)^n}$,
d) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n!}$, e) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{n}{4^n}$, f) $\sum\limits_{n=2}^{\infty} (-1)^n \frac{\ln n}{n \ln(\ln n)}$.

Answers: yes – (b), (c), (d), (e), no – (a), (f).

Exercise 3. Using the fact "absolute convergence ⇒ convergence" justify the fact, that these series are convergent.

a) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{n+1}{n^3+n}$, b) $\sum\limits_{n=1}^{\infty} \frac{\sin(\frac{\pi}{2}+n\pi)}{n^2+1}$, c) $\sum\limits_{n=1}^{\infty} (-1)^n \frac{n}{3^n}$, d) $\sum\limits_{n=2}^{\infty} \frac{\cos(n\pi)}{n \ln^2 n}$.

Exercise 4. Establish the type of convergence (absolute or conditional) of these series.

a) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$, b) $\sum\limits_{n=1}^{\infty} \frac{\cos n\pi}{n}$, c) $\sum\limits_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$, d) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$, e) $\sum\limits_{n=2}^{\infty} \frac{(-1)^n}{\ln^2 n}$.

Answers: absolute – (a), conditional – (b), (c), (d), (e).

Nobody's perfect – if you notice any mistakes in answers, please let us know!