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Extremes of 2-variable functions:

$f(x,y) = xy(x+y-6) = x^2y + xy^2 - 6xy$

I Partial derivatives:

$f_x = 2xy + y^2 - 6y \qquad f_{xx} = 2y$

$f_y = x^2 + 2xy - 6x \qquad f_{yy} = 2x$

$f_{xy} = (2xy + y^2 - 6y)'_y = 2x + 2y - 6$

II Critical (= stationary) points

$$\begin{cases} 2xy + y^2 - 6y = 0 \\ x^2 + 2xy - 6x = 0 \end{cases} \Rightarrow \quad y(2x+y-6) = 0$$

$$y = 0 \quad \vee \quad 2x + y - 6 = 0$$

a) $y = 0$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0 \quad \vee \quad x = 6$

$P_1(0,0) \quad P_2(6,0)$

b) $2x + y - 6 = 0$

$y = -2x + 6$

$x^2 + 2x(-2x+6) - 6x = 0$

$x^2 - 4x^2 + 12x - 6x = 0$

$-3x^2 + 6x = 0$

$-3x(x-2) = 0$

$x = 0 \quad \vee \quad x = 2$

$P_3(0,6) \quad P_4(2,2)$

III Determinants

$$D_f = \begin{vmatrix} 2y & 2x+2y-6 \\ 2x+2y-6 & 2x \end{vmatrix}$$

$D_f(0,0) = \begin{vmatrix} 0 & -6 \\ -6 & 0 \end{vmatrix} = -36 < 0$, saddle point

$D_f(0,6) = \begin{vmatrix} 12 & 6 \\ 6 & 0 \end{vmatrix} = -36 < 0$, saddle point

$D_f(6,0) = \begin{vmatrix} 0 & 6 \\ 6 & 12 \end{vmatrix} = -36 < 0$, saddle point

$D_f(2,2) = \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} = 12 > 0$, $(2y)|_{(2,2)} = 4 > 0$ minimum

IV Answer

$f(2,2) = -8$ is a minimum

Saddle points:

$f(0,0) = 0 \qquad P = (0,0,0)$

$f(0,6) = 0 \qquad P = (0,6,0)$

$f(6,0) = 0 \qquad P = (6,0,0)$

WolframAlpha computational knowledge engine

plot x^2*y+x*y^2-6*x*y,x=1.99..2.01 y=2.01..1.99

Input interpretation:

plot	$x^2\,y + x\,y^2 - 6\,x\,y$	$x = 1.99$ to 2.01
		$y = 2.01$ to 1.99

3D plot:

Show contour lines