

A.L. gr 3. Homework 2

$$f(x, y) = x^2 - 2xy + 2y^2 - 2y$$

$$\frac{\partial f}{\partial x} = 2x - 2y \quad \frac{\partial^2 f}{\partial x^2} = 2 \quad \frac{\partial^2 f}{\partial x \partial y} = (2x - 2y)'_x = 2$$

$$\frac{\partial f}{\partial y} = -2x + 4y - 2 \quad \frac{\partial^2 f}{\partial y^2} = 4$$

critical points:

$$\begin{cases} 2x - 2y = 0 \\ -2x + 4y - 2 = 0 \end{cases} \quad \begin{matrix} | :2 \\ \Rightarrow \end{matrix} \begin{cases} x - y = 0 \\ -x + 2y - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = y \\ -y + 2y = 1 \end{cases} \Rightarrow \begin{cases} x = y \\ y = 1 \end{cases}$$

determinant:

$$D_f = \begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix} = 4$$

$D_{(1,1)} > 0$ and $\frac{\partial^2 f}{\partial x^2}(1,1) > 0$, then f has a local minimum at $(1,1)$

answer:

$$f(1,1) = 2 \cdot 1^2 - 2 \cdot 1 \cdot 1 + 2 \cdot 1^2 - 2 \cdot 1 = 2 - 2 + 2 - 2 = 0$$

is a minimum



plot $x^2 - 2xy + 2y^2 - 2y$, $x=0.99..1.01$, $y=0.99..1.01$

input interpretation:

plot

$$x^2 - 2xy + 2y^2 - 2y$$

$x = 0.99$ to 1.01

$y = 0.99$ to 1.01

3D plot:

