Extremes of 2-variable functions

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$$f(x,y) = 4xy - x^4 - y^4$$

I Partial derivatives

$$\frac{\partial f}{\partial x} = (4xy - x^4 - y^4)'_x = 4y - 4x^3$$

$$\frac{\partial f}{\partial y} = (4xy - x^4 - y^4)'_y = 4x - 4y^3$$

$$\frac{\partial^2 f}{\partial x^2} = (4y - 4x^3)'_x = -12x^2$$

$$\frac{\partial^2 f}{\partial y^2} = (4x - 4y^3)'_y = -12y^2$$

$$\frac{\partial^2 f}{\partial x \partial y} = (4x - 4y^3)'_{\substack{y \\ x = const}} = 4$$

II Critical (= stationary) points

$$\begin{cases} 4y - 4x^3 = 0 \\ 4x - 4y^3 = 0 \end{cases} \Rightarrow \quad 4y = 4x^3 \; /:4 \quad y = x^3 \quad \Leftrightarrow \quad \begin{cases} y = x^3 \\ 4x - 4(x^3)^3 = 0 \end{cases}$$

$$4x - 4x^9 = 0 \; /:4$$

$$x - x^9 = 0$$

$$x(1 - x^8) = 0$$

$$1 = x^8$$

$$x = 1 \quad \vee \quad x = (-1)$$

$x_1 = 0$	$y_1 = 0$	$P_1 = (0,0)$
$x_2 = 1$	$y_2 = 1$	$P_2 = (1,1)$
$x_3 = -1$	$y_3 = -1$	$P_3 = (-1,-1)$

III Determinants

$$D_f = \begin{vmatrix} -12x^2 & 4 \\ 4 & -12y^2 \end{vmatrix} = 144x^2y^2 - 16$$

$D_f(0,0) = 144 \cdot 0^2 \cdot 0^2 - 16 = -16 < 0$ saddle point

$D_f(1,1) = 144 \cdot 1^2 \cdot 1^2 - 16 = 128$

$$\frac{\partial^2 f}{\partial x^2} = -12x^2 = -12 < 0 \quad \text{Maximum}$$

$$f_{max}(1,1) = 4 \cdot 1 \cdot 1 - 1^4 - 1^4 = 2$$

$D_f(-1,-1) = 144 \cdot (-1)^2 \cdot (-1)^2 - 16 = 128 > 0$

$$\frac{\partial^2 f}{\partial x^2} = -12x^2 = -12 < 0 \quad \text{Maximum}$$

$$f_{max}(-1,-1) = 4 \cdot (-1) \cdot (-1) - (-1)^4 - (-1)^4 = 2$$

IV Answer

$f_{max}(1,1) = 2$

$f_{max}(-1,-1) = 2$

saddle point $(0,0)$

plot4*x*y−x^4−y^4,x=1..−1,y=1..−1

Input interpretation:

plot	$4xy-x^4-y^4$	$x = 1$ to -1 $y = 1$ to -1

3D plot: