Before solving the following exercises, you should revise the following material:

- Basic calculus → Derivatives → Derivatives - basics
- Basic calculus → Derivatives → Exercises - derivatives

Exercise 1. Using the definition, calculate all partial derivatives at point P. Verify your results by calculating the same derivatives using derivation formulas.

a) $f(x,y)=\frac{x}{y},\ P=(-1,1)$, b) $f(x,y)=y\sin x,\ P=(0,\pi)$,
c) $f(x,y)=\sqrt[3]{xy},\ P=(0,0)$, d) $f(x,y,z)=x+2xy-3xyz,\ P=(1,2,3)$.

Exercise 2. Check that the following functions are continuous in given points, but they do not have partial derivatives there.

a) $f(x,y)=\sqrt{x^2+y^2},\ P=(0,0)$, b) $f(x,y)=|x|+|y-1|,\ P=(0,1)$.

Exercise 3. Calculate the first order partial derivatives of the following functions.

$a(x,y)=x^2+xy+y^2+x^3+y^3+(xy)^2$, $b(x,y,z)=xy\sqrt{z}+yz\sqrt{x}+zx\sqrt{y}$,
$c(x,y)=\ln(x^2+y^2)$, $d(x,y,z)=(\frac{y}{x})^z$,
$e(x,y)=x^y+y^x+5$, $f(x,y,z)=xy^2z^3+e^{sin(x^3y^2z)}+x^2-y^3+z-7$,
$g(x,y,z)=\cos^3(5x-y^3+z)+\ln(z\ln xy)$, $h(x,y)=\operatorname{arctg}(y\sqrt{x})+\sin^2(3x^2+xy-5y^3)$.

Exercise 4. Calculate all second order partial derivatives of functions from **Exercise 3**.

Exercise 5. Calculate all second order partial derivatives of the following functions.

$a(x,y)=\ln(4x^2+2y^4+1)$, $b(x,y)=ye^{xy}$, $c(x,y,z)=z\cos(x^2+y^2)$,
$d(x,y)=x\sin(x+y)+e^y$, $e(x,y)=(x-y)e^{3x+5y}$, $f(x,y)=x^y$.

Exercise 6. Check if function u satisfies given equations.

a) $u(x,y)=x^yy^x$, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=(x+y+\ln u)u$,

b) $u(x,y)=\ln(e^x+e^y)$, $\frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial^2 u}{\partial y^2}=(\frac{\partial^2 u}{\partial x\partial y})^2$,

c) $u(x,y)=\ln x\ln y$, $\frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial^2 u}{\partial y^2}-u(\frac{\partial^2 u}{\partial x\partial y})^2=0$,

d) $u(x,y)=2\cos^2(y-\frac{x}{2})$, $2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}=0$,

e) $u(x,y)=x\sin y+y\sin x$, $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-u$,

f) $u(x,y)=xe^y+ye^x$, $u_{xxx}+u_{yyy}=xu_{xyy}+yu_{xxy}$.

Exercise 7. Write equations of planes tangent to graphs of the following functions at given points.

$a(x,y) = x^y, \quad P = (2,4,16),$

$b(x,y) = y\ln(2 + x^2y - y^2), \quad P = (2,1,b(2,1)),$

$c(x,y) = \frac{\arcsin x}{\arcsin y}, \quad P = (-\frac{1}{2}, \frac{\sqrt{3}}{2}, -1),$

$d(x,y) = e^{x\cos y}, \quad P = (1,\pi,\frac{1}{e}),$

$e(x,y) = \sin x\cos x, \quad P = (\frac{\pi}{4}, \frac{\pi}{4}, \frac{1}{2}).$

Exercise 8. Using the total differentials, calculate the approximated values of the following expressions.

a) $\sqrt[3]{(2.06)^2 + (1.97)^2}$, b) $0.98\ln 1.01$, c) $(1.03)^{3.01}$, d) $\arctan\frac{0.02}{1.99}$,
e) $\sqrt{(1.06)^2 + (1.97)^3}$, f) $(1.95)^2e^{0.02}$, g) $\frac{(1.01)^3-(2.99)^2}{(1.01)^3+(2.99)^2}$, h) $\ln(\sqrt{1.04} + \sqrt[4]{0.96} - 1)$.