PART I — Basic properties of a sequence.

Exercise 1. Calculate: $2!$, $4!$, $2!\cdot 3!$, $(2\cdot 3)!$, $\binom{4}{1}$, $\binom{3}{2}$.
Exercise 2. Simplify: $\frac{(n-1)!}{(n+1)!}$, $\frac{(n+2)!(n+1)!}{n!\cdot(n-1)!}$, $\frac{(n+1)!+n!}{(n+1)!-n!}$, $\frac{\binom{n+2}{n}}{n^2}$.
Exercise 3. Write down the first five terms of each sequence and check the monotonicity. In each case state whether the sequence is bounded.

$$a_n = 2n^2 - 3n + 1, \quad b_n = \frac{4n+5}{2n+1}, \quad c_n = n^2 - n - 1,$$
$$d_n = \frac{n^2-1}{n}, \quad e_n = \left(\frac{2}{3}\right)^n, \quad f_n = -\frac{1}{2}\cdot 4^{n-1},$$
$$g_n = \frac{2^n}{n!}, \quad h_n = (-1)^n \cdot \frac{1}{n^2}, \quad i_n = \frac{1+n^2}{1+n^3}.$$

PART II — Arithmetic and geometric sequences.

Exercise 4. We know that $a_1 = 6$ and $a_8 = 34$. Find $a_2, \ldots, a_7$ so that (a_n) becomes an arithmetic sequence.
Exercise 5. The sum of x first terms of a sequence $a_n = 5n + 2$ is 553. Calculate x.
Exercise 6. Calculate the number of terms in a sum $2 + 5 + 8 + 13 + \cdots + 449$, calculate the sum as well.
Exercise 7. Calculate the sum of all two-digit numbers.
Exercise 8. Check if numbers: $\sqrt{5}$, $\frac{\sqrt{5}}{\sqrt{5}-2}$, $\frac{5+2\sqrt{5}}{\sqrt{5}-2}$ given in that specific order create a geometric sequence.
Exercise 9. Find the first term of a geometric sequence in which the common ratio is 2 and the sum of eight first terms is 765.
Exercise 10. Find the first term and the common difference of an arithmetic sequence knowing that:

a) $a_7 - a_3 = 8$ and $a_2 \cdot a_7 = 75$, b) $a_2 + a_5 - a_3 = 10$ and $a_2 + a_9 = 17$.

Exercise 11. Find the first term of a geometric sequence knowing that:

a) $q = -\sqrt{2}$ and $S_6 = -7$, b) $q = \frac{2}{3}$ and $S_4 = 65$.

Exercise 12. Check if (a_n) is a geometric progression if: a) $a_n = 3 \cdot 2^{4n+1}$, b) $a_n = 3^{n+1} \cdot 2^n - 6^n$.

PART III — The limit of a sequence.

Exercise 13. Draw graphs of the following sequences and find their limits (if they exist).

$$a_n = n - 1, \quad b_n = \frac{1}{n}, \quad c_n = \frac{1}{n} + 2, \quad d_n = \frac{(-1)^{n+1}}{n}, \quad e_n = \frac{n-1}{n}, \quad f_n = (-1)^n \frac{n-1}{n}, \quad g_n = 3.$$

Exercise 14. Give your own two examples of a divergent sequence.
Exercise 15. Calculate the limits of the following sequences.
Hints:
$(a_n \to d_n)$ and $(i_n \to l_n)$ – basic operations on limits,
$(e_n \to h_n)$ – use $\lim_{n\to\infty} q^n$,
$(m_n \to s_n)$ – sandwich theorem or "two sequences" theorem,
$(t_n \to y_n)$ – use $\lim_{n\to\infty}(1 + \frac{1}{b_n})^{b_n} = e$.

$a_n = \frac{4n^5-6n^2+1}{6n^5+n^3-8}$, $b_n = \frac{-4n^2+n-9}{3n^6+2n-1}$, $c_n = \frac{\binom{n+2}{n}}{n^2}$, $d_n = \frac{3n^6+2n-1}{-4n^2+n-9}$,
$e_n = \frac{5\cdot 9^n-5^{n+1}+3}{-3^{2n}+2^{3n+1}-1}$, $f_n = \frac{5^{n+1}-2\cdot 3^n+8}{2^{3n+1}-5^n+7}$, $g_n = \frac{-3\cdot 2^{3n}-2^n+5}{7\cdot 3^{n+2}-2^{2n+1}+1}$, $h_n = \sqrt{\pi^n} - \sqrt{e^n}$,
$i_n = \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}$, $j_n = \frac{\sqrt[3]{n^2+1}-1}{\sqrt[3]{n^2-n}}$, $k_n = \sqrt{n^2+1} - \sqrt{n^2-1}$, $l_n = \sqrt[3]{n^3+2n^2} - n$,
$m_n = \sqrt[n]{e^n+\pi^n+8^n}$, $o_n = \sqrt[n]{7+\cos(n\pi)}$, $p_n = \frac{5\cdot 4^n+3\sin(n!)}{2^{2n}+7}$, $q_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}$,
$r_n = (4-\arctan n)^n$, $s_n = \frac{2n^3+1}{4n^2+\cos(n^2)}$, $t_n = (\frac{n+5}{n})^{3n}$, $u_n = (\frac{n-2}{n+4})^{5n+2009}$,
$v_n = (\frac{n^2+n}{n^2-3n-4})^{n-10}$, $w_n = (1-\frac{1}{n^2})^n$, $x_n = (\frac{n+4}{n+3})^{1979-2n}$, $y_n = (1-\frac{2}{n^2})^{2-3n}$.

Exercise 16. Calculate the following limits. In each case firstly simplify the n-th term.

$$a_n = \frac{1}{n} + \frac{2}{n} + \cdots + \frac{n-1}{n}, \quad b_n = \frac{1+2+\cdots+2n}{(2-3n)(n+2)}, \quad c_n = \frac{1}{n^3} + \frac{2}{n^3} + \cdots + \frac{n-1}{n^3}.$$

Exercise 17. Calculate sums of the following infinite geometric sequences.

$$\text{a) } 7 + 2.1 + 0.63 + \ldots, \quad \text{b) } \sqrt{5} + \frac{1}{\sqrt{5}} + \frac{1}{5\sqrt{5}} + \ldots.$$

Exercise 18. Express the following periodic decimal fractions as common fractions: a) 3.(14), b) 0.3(21), c) 1.24(36).

Most exercises were taken from the script "Matematyka - podstawy z elementami matematyki wyższej" issued by the Gdańsk University of Technology publishing house.