

Exercise 1. Calculate $P(x) + Q(x)$, $P(x) - Q(x)$ and $P(x) \cdot Q(x)$.

$$\text{a) } P(x) = 4 - x^2, Q(x) = 1 + 2x^3, \quad \text{b) } P(x) = 1 - x^3, Q(x) = -1 + 5x + x^2.$$

Exercise 2. Revise the short multiplication formulas.

$$\begin{aligned} \text{a) } (a - b)^2 = \dots, \quad \text{b) } (a + b)^2 = \dots, \quad \text{c) } (a - b)(a + b) = \dots, \quad \text{d) } (a + b + c)^2 = \dots, \\ \text{e) } (a + b)^3 = \dots, \quad \text{e) } (a - b)^3 = \dots, \quad \text{f) } a^3 + b^3 = \dots, \quad \text{g) } a^3 - b^3 = \dots \end{aligned}$$

Exercise 3. Find zeroes of the following functions and draw graphs.

$$\text{a) } f(x) = 2x + 1, \text{ draw } f(x) \text{ and } |f(-|x|)|,$$

$$\text{b) } f(x) = \frac{x}{3} + 2, \text{ draw } f(x), |f(x)| \text{ and } f(|x|),$$

$$\text{c) } f(x) = 4, \text{ draw } f(x) \text{ and } f(x + 2) + 3.$$

Exercise 4. For each parabola calculate its roots (if they exist) and coordinates of the tip. Draw graphs – mark roots, coordinates of the tip and the intersection with OY axis on each graph.

$$\text{a) } f(x) = 3x^2 + 4, \text{ draw } f(x),$$

$$\text{b) } f(x) = x^2 - 9x + 8, \text{ draw } f(x) \text{ and } f(|x|),$$

$$\text{c) } f(x) = -2x^2 - 10x + 12, \text{ draw } f(x) \text{ and } |f(x + 2)| + 3.$$

Exercise 5. Calculate the canonical form and roots without computing the discriminant. Draw a graph of each parabola and mark all important points.

$$\text{a) } f(x) = -2x^2 - 2x + 24, \quad \text{b) } f(x) = x^2 - 7x + 6, \quad \text{c) } f(x) = 3x^2 + 9x - 120.$$

Exercise 6. Find values of parameter m for which the quadratic function $y = -x^2 + (m + 2)x - 4$ has

$$\text{a) no roots,} \quad \text{b) exactly one root,} \quad \text{c) two roots.}$$

Exercise 7. Solve the following biquadratic equations.

$$\text{a) } x^4 - 3(x^2 - 1) = 7(x^2 - 3), \quad \text{b) } x^4 + x^2 - 6 = 0.$$

Exercise 8. Divide polynomials.

$$\begin{aligned} \text{a) } (x^2 - 9x - 10) : (x + 1), \quad \text{b) } (x^2 + 9x + 14) : (x + 7), \quad \text{c) } (3x^3 - 5x^2 + 10x - 3) : (3x + 1), \\ \text{d) } (2x^4 - 5x^3 + 2x) : (2x + 1), \quad \text{e) } (x^{15} - 1) : (x^5 + 1). \end{aligned}$$

Exercise 9. Find all roots of the following polynomials.

$$\text{a) } x^3 - 2x^2 - 5x + 6, \quad \text{b) } 2x^3 - 5x^2 - 2x - 3, \quad \text{c) } x^3 + \frac{x^2}{6} - x + \frac{1}{3}.$$

Exercise 10. Find roots of the polynomials and solve inequalities. You may not “guess” any roots in examples (a) and (c).

$$\text{a) } 2x^3 - x^2 - 4x + 2 < 0, \quad \text{b) } -2x^3 - 2x^2 + 5x + 2 > 0, \quad \text{c) } -16x^6 + 8x^5 - 4x^4 + 2x^3 \geq 0.$$

Exercise 11. Sketch the following curves.

$$\text{a) } y = 2 + \frac{3}{x-4}, \quad \text{b) } y = \frac{-5}{x+5} - 2, \quad \text{c) } y = \frac{x-1}{x+1}, \quad \text{d) } y = -\frac{x-3}{x+2} + 3.$$

Exercise 12. Solve the following inequalities.

$$\text{a) } \frac{1}{x+2} - \frac{x+2}{x-2} \leq 2, \quad \text{b) } \frac{6}{3+5x} \geq 3, \quad \text{c) } \frac{-7}{9-2x} \geq \frac{5}{8x+6}, \quad \text{d) } \frac{1+x}{1+2x} - \frac{1-2x}{x+1} < -1.$$

Exercise 13. Set domains of the following functions.

$$\text{a) } f(x) = (x+1)^{-5} + (2-x)^{-1.3}, \quad \text{b) } g(x) = x(x-3)^{7/3} + x^4(7-x)^{1/2}, \quad \text{c) } h(x) = x^2(x-2)(5-x)^{\sqrt{\pi}}.$$

Exercise 14. Set domains of the following functions.

$$\begin{aligned} \text{a) } f(x) &= \sqrt{2x-x^2}, & \text{b) } f(x) &= \frac{1}{\sqrt[3]{x^6-1}}, & \text{c) } f(x) &= \sqrt[4]{x-\frac{1}{x}}, \\ \text{d) } f(x) &= \sqrt{\frac{3}{x}-2-x}, & \text{e) } f(x) &= \sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}. \end{aligned}$$

Exercise 15. Solve the following equations and inequalities.

$$\begin{aligned} \text{a) } \sqrt{10x+8} &= 8-x, & \text{b) } \sqrt[5]{x-3} &< -2, & \text{c) } \sqrt{x-3} &< -2, \\ \text{d) } \sqrt{6-x} &> 7, & \text{e) } \sqrt{6-x} &> -7, & \text{f) } \sqrt{12-x^2} &> 5x-7, \\ \text{g) } (x^2+x)^{-2.5} &= 0.25\sqrt{8}, & \text{h) } \sqrt{4x-x^2} &> x-2. \end{aligned}$$