

Exercise 1. Solve the following equations and inequalities.

$$\begin{aligned} \text{a) } (0.25)^{3x-1} &= (\sqrt{2})^{5-x}, & \text{b) } 16^{\frac{x}{x+3}} &= 4\left(\frac{2^x}{8}\right)^{\frac{1}{2x+5}}, & \text{c) } 8^{x-2} &< 2^{4-2x}, \\ \text{d) } (0.3)^{7-x} &\leq (0.09)^{x+2}, & \text{e) } 7^{-x} - 3 \cdot 7^{x+1} &> 4, & \text{f) } 2^{2x+1} - 33 \cdot 2^{x-1} + 4 &= 0. \end{aligned}$$

Exercise 2. Find intersection points of graphs of the following pairs of functions.

$$\text{a) } y = 8^{x^2 - \frac{2}{3}}, y = 0.125^{x + \frac{2}{3}}, \quad \text{b) } y = 2^{x^2 + 5x + \frac{9}{2}}, y = 0.25^{-x^2 - 4.5x - 3.75}.$$

Exercise 3. Solve the following inequalities.

$$\begin{aligned} \text{a) } \left(\frac{9}{11}\right)^{9x^2 - 11x} &\geq \left(\frac{11}{9}\right)^{11x^2 - 9x}, & \text{b) } 5 \cdot 4^x + 2 \cdot 25^x &\geq 7 \cdot 10^x, \\ \text{c) } 5^x - 20 &> 5^{3-x}, & \text{d*) } 3^{2x} + \left(\frac{1}{2}\right)^{-x} \cdot 3^{x+1} - 2^{2x+2} &\geq 0. \end{aligned}$$

Exercise 4. Calculate exact values.

$$\begin{aligned} \text{a) } \log_2 4, & \quad \text{b) } \log_2 2, & \text{c) } \log_3 3, & \quad \text{d) } \log 1, & \quad \text{e) } \log_{0.5} 1, & \quad \text{f) } \log_6 1, \\ \text{g) } 2^{\log_2 5}, & \quad \text{h) } 3^{\log_3 5}, & \text{i) } \log_4 4^5, & \quad \text{j) } \log 10^5. \end{aligned}$$

Exercise 5. Solve the following equations.

$$\begin{aligned} \text{a) } \log(3-x)(x-5) &= \log(x-3) + \log(5-x), & \text{b) } \log_3(4 \cdot 3^{x-1} - 1) &= 2x - 1, \\ \text{c) } \log(5x^2 + 2x - 1) - \log(x+2) &= 1. \end{aligned}$$

Exercise 6. Solve the following inequalities.

$$\begin{aligned} \text{a) } \log(x-4) + \log x &\leq \log 21, & \text{b) } 3^{\log_3^2 x} + x^{\log_3 x} &\leq 162, \\ \text{c) } \log(2^x + x - 13) &> x - x \log 5, & \text{d) } \log_{(x-2)} \frac{x-1}{x-3} &\geq 1. \end{aligned}$$

Exercise 7. Which number is bigger?

$$\text{a) } \log_3 222 \text{ or } \log_2 33, \quad \text{b) } \pi^e \text{ or } e^\pi.$$

Most exercises were taken from the script "Matematyka - podstawy z elementami matematyki wyższej" issued by the Gdańsk University of Technology publishing house.

VOCABULARY - EXPONENTIALS AND LOGARITHMS

- exponential function - funkcja wykładnicza
- exponent - wykładnik
- logarithmic function - funkcja logarytmiczna
- logarithm - logarytm
- $\log_a x$ - the logarithm of x to the base a
- $\log_3 x$ - the logarithm of x to third base
- $\log x$ - decimal logarithm of x
- $\ln x$ - natural logarithm of x