

How to calculate complex roots

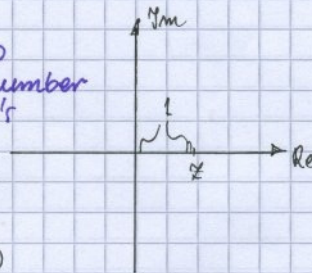
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Example 1° $\sqrt[4]{1}$ a fourth root of 1

$$z = 1$$

$$n = 4$$

I have to draw my number to find its angle



It's easy to notice that: $\varphi = 0$

I also have to calculate the modulus of z :

$$|z| = \sqrt{1^2} = 1$$

I expect to find four roots: z_0, z_1 and z_2, z_3

$$z_0 = \sqrt[4]{1} \cdot \left(\cos \frac{\varphi + 2\pi \cdot 0}{4} + i \sin \frac{\varphi + 2\pi \cdot 0}{4} \right)$$

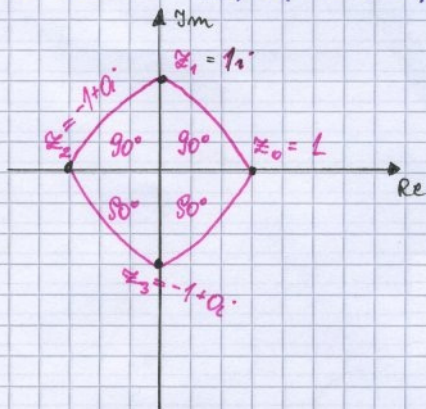
because I expect to find 4 roots

because it's the root number 0

$$\text{So: } z_0 = \sqrt[4]{1} \cdot \left(\cos \frac{0 + 2\pi \cdot 0}{4} + i \sin \frac{0 + 2\pi \cdot 0}{4} \right) = 1(\cos 0 + i \sin 0) = 1 (+0i)$$

$$z_1 = \sqrt[4]{1} \left(\cos \frac{0 + 2\pi \cdot 1}{4} + i \sin \frac{0 + 2\pi \cdot 1}{4} \right) = 1 \left(\underbrace{\cos \frac{\pi}{2}}_0 + i \underbrace{\sin \frac{\pi}{2}}_1 \right) = 1i$$

I will draw z_0, z_1 and find next angles.



I will have 4 roots and I divide the circle into 4 rotated parts.

$$\frac{360^\circ}{4} = 90^\circ$$

My Answer: $\sqrt[4]{1} \in \{1; 1i; -1+0i; 0-1i\}$