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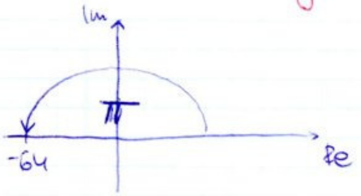
EPM, 2nd sem

Example $\sqrt[6]{-64}$

$z = -64$ my number z is equal to -64 (or $-64+0i$)

$z = -64 + 0i$

I have to draw my number to find its angle.



It's easy to notice that: $\varphi = \pi$ $(=180^\circ)$

I also have to calculate the modulus of z:

$$|z| = \sqrt{(-64)^2 + 0^2} = 64$$

I expect to find six roots: $z_0, z_1, z_2, z_3, z_4, z_5$

The zeroth root can be calculated from the following formula:

$$z_0 = \sqrt[6]{|z|}\left(\cos\frac{\varphi + 2\pi\cdot 0}{6} + i\sin\frac{\varphi+2\pi\cdot 0}{6}\right)$$

because I expect to find 6 roots

because it's the root number 0

So: $z_0 = \sqrt[6]{64}\left(\cos\frac{\pi+2\pi\cdot 0}{6} + i\sin\frac{\pi+2\pi\cdot 0}{6}\right) = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) =$

$= 2\cdot\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2}i = \sqrt{3} + i$ $\qquad \sqrt{3} \approx 1{,}73\ldots$

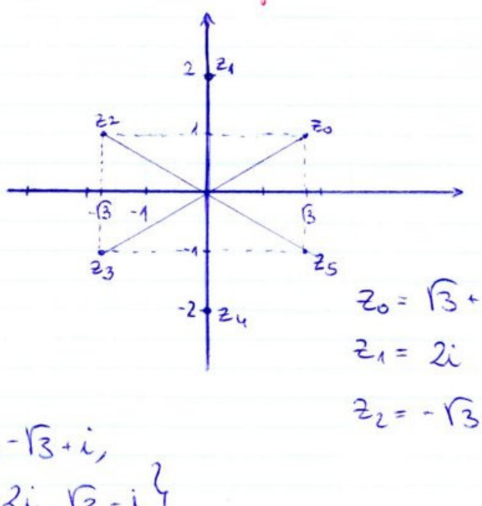
$z_1 = \sqrt[6]{64}\left(\cos\frac{\pi+2\pi\cdot 1}{6} + i\sin\frac{\pi+2\pi\cdot 1}{6}\right) = 2\left(\cos\frac{3\pi}{6} + i\sin\frac{\pi}{2}\right) =$

$= 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2(0 + 1i) = 2i$

$z_2 = \sqrt[6]{64}\left(\cos\frac{\pi+2\pi\cdot 2}{6} + i\sin\frac{\pi+2\pi\cdot 2}{6}\right) = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) =$

$= 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$

I will draw z_0, z_1 and z_2 and then "guess" other roots:



$z_0 = \sqrt{3} + i$ $\quad z_3 = -\sqrt{3} - i$

$z_1 = 2i$ $\quad z_4 = -2i$

$z_2 = -\sqrt{3} + i$ $\quad z_5 = \sqrt{3} - i$

Answer: $\sqrt[6]{-64} \in \{\sqrt{3}+i, 2i, -\sqrt{3}+i, -\sqrt{3}-i, -2i, \sqrt{3}-i\}$