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EPM, 3 gr.

Extra Homework for volunteers

1. Definition

If we have a n-root of a complex number that exist "n" complex numbers, which, when raised to the power "n", give us a number z.

2. First we calculate modulus

$|z| = \sqrt{a^2 + b^2}$

The modulus is equel squere root of the real part of z squared, plus the imaginary part of z squared.

3. Formula

A root of any degree is described by this formula

$$\sqrt[n]{z} = z_k \quad , k = 0, 1, 2, \dots, n-1$$

$$z_k = \sqrt[n]{|z|} \cdot \left[\cos\left(\frac{\varphi + 2\pi \cdot k}{n}\right) + i\sin\left(\frac{\varphi + 2\pi \cdot k}{n}\right) \right]$$

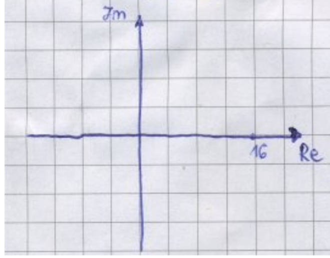
4. Example

$\sqrt[4]{16}$

$z = 16$ (number under the root)
$n = 4$ (number's of roots)

$Re(z) = 16$
$Im(z) = 0$

5. Find the beginning angle $\varphi = 0$



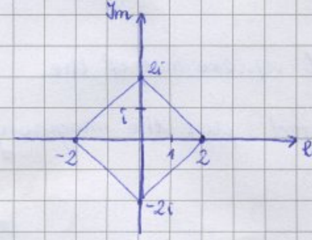
6. Calculation

$$z_0 = \sqrt[4]{16} \cdot \left(\cos\left(\frac{0 + 2\pi \cdot 0}{4}\right) + i\sin\left(\frac{0 + 2\pi \cdot 0}{4}\right)\right) = \sqrt[4]{16}\,(1 + 0i) = 2$$

$$z_1 = \sqrt[4]{16} \cdot \left(\cos\left(\frac{0 + 2\pi \cdot 1}{4}\right) + i\sin\left(\frac{0 + 2\pi \cdot 1}{4}\right)\right) = \sqrt[4]{16}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) =$$
$$= \sqrt[4]{16} \cdot (0 + 1i) = 2i$$

$$z_2 = \sqrt[4]{16} \cdot \left(\cos\left(\frac{0 + 2\pi \cdot 2}{4}\right) + i\sin\left(\frac{0 + 2\pi \cdot 2}{4}\right)\right) = \sqrt[4]{16}\,(\cos\pi + i\sin\pi) = \sqrt[4]{16}\,(-1 + 0i) =$$
$$= -2$$

$$z_3 = \sqrt[4]{16} \cdot \left(\cos\left(\frac{0 + 2\pi \cdot 3}{4}\right) + i\sin\left(\frac{0 + 2\pi \cdot 3}{4}\right)\right) = \sqrt[4]{16} \cdot (0 + (-1)i) = -2i$$



n-roots of z are a set of points distante from the center of coordinate system up $\sqrt[n]{z}$ and shifted relative to each other on angle $\frac{2\pi}{n}$. In my exemple the angle is equel 90 degrees.