How to invert matrices using the Gaussian elimination algorithm

Example: $A = \begin{bmatrix} 0 & 1 & 1 \\ -4 & -2 & 1 \\ 3 & 3 & 1 \end{bmatrix}$

First of all, I rewrite my matrix, then follow it by a vertical line and a unit matrix of appropriate dimensions.

$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ -4 & -2 & 1 & 0 & 1 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$ My goal is to perform a certain number of operations that will produce a matrix: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right]$ (I_3)

It's very comfortable to have one in the upper left corner. First I will exchange two rows and then I will have here 1.

$r_1 \leftrightarrow r_2$ → $\left[\begin{array}{ccc|ccc} -4 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$ $r_1 \to r_1 + r_3$ →

Now, to get 1 I can divide first row by -4, however, it will produce fractions, or I can add row #3 to the row #1.

$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$ $r_1 \cdot (-1)$ → $\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$ $r_3 \to r_3 - 3r_1$ →

I get -1 but I want to have 1 so I multiply row 1 by -1

Now, I would like to get rid of this number to have perfect I_3 first column. I substract from #row 3 times #1 row

$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 6 & 7 & 0 & 3 & 4 \end{array}\right]$ $r_2 \to r_2 + r_1$ → $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 6 & 7 & 0 & 3 & 4 \end{array}\right]$ $r_3 \to r_3 - 6r_2$ → $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & 3 & 4 \end{array}\right]$ $r_1 \to r_1 + r_3$ →

The first column looks exactly like in I_3. I would like to have 0 here.

I want to have 0 here so I need to substract 6 times #2 row

I want to get rid of -1, so I can simply add #3 row.

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 3 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & 3 & 4 \end{array}\right]$ $r_2 \to r_2 - r_3$ → $\left[\begin{array}{c|ccc} I_3 & -5 & 2 & 3 \\ & 7 & -3 & -4 \\ & -6 & 3 & 4 \end{array}\right]$ $= A^{-1}$

If I get rid of this number I will have matrix I_3 in front of the line

Author: Natalia Korcz
(EPM, II semester)