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$$B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

I have to find B^{-1} . So I've written a unit matrix I_4 next to the matrix B .

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

\downarrow B \downarrow I_4

Now I want to exchange my matrix B into I_4 . To do this I have to remove 1 in 1 column and 3 row. So I want to (subtract) $r_3 - r_1$.

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 - r_1} \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & -1 & -2 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 - 2r_2} \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -4 & -1 & -4 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 - 3r_2} \left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

Now I want to remove 2 from 2 column and 3 row.
Next I change 3 to 0 from 2 column and 4 row.

$$\xrightarrow{r_1 - r_2 \rightarrow r_1} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 2 & -5 & 0 & -3 & 0 & 1 \end{array} \right] \xrightarrow{r_4 \rightarrow r_4 - 2r_3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right] \xrightarrow{r_1 - r_3 \rightarrow r_1} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 8 & 2 & 6 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right]$$

So now my first 2 columns in matrix B are the same as the first two columns in a unit matrix I_4 . To have the same 3 column as 3 column of a unit matrix I have to exchange 1 from 3 column and 1 row and 2 from 3 column and 4 row into zeros. To get 4 column the same as 4 column of the unit matrix I want to have only at the 4 column and 4 row 1 and on the other row zeros.

$$\xrightarrow{r_1 \div 3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 8 & 2 & 6 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right] \xrightarrow{r_1 \rightarrow r_1 - 2r_2} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 2 & 4 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right] \xrightarrow{r_2 \rightarrow r_2 - 2r_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 2 & 4 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 2 & 4 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -10 & -1 & -7 & 1 & 0 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 + 4r_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 2 & 4 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 10 & 7 & 9 & -3 & 4 \\ 0 & 0 & 0 & 5 & 2 & 5 & -1 & 1 \end{array} \right]$$

at matrix B which I exchanged at unit matrix I_4 here I get B^{-1} .