Homework III

"How to use Gaussian elimination to solve a system of equation."

EXAMPLE:
$$\begin{cases} 2x + y + z = 8 \\ 3x - 2y - z = 1 \\ 4x - 7y + 3z = 10 \end{cases}$$

I can rewrite the above system of equations into three matrices:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -2 & -1 \\ 4 & -7 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 8 \\ 1 \\ 10 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now, I have to transform my matrix [A/B] into $\left[I_3 \,\middle|\, \begin{matrix} x \\ y \\ z \end{matrix} \right]$ by using elementary operations on matrices.

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 8 \\ 3 & -2 & -1 & 1 \\ 4 & -7 & 3 & 10 \end{array}\right] \xrightarrow{r_3 = r_3 - r_2} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 8 \\ 3 & -2 & -1 & 1 \\ 1 & -5 & 4 & 9 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{ccc|c} 1 & -5 & 4 & 9 \\ 3 & -2 & -1 & 1 \\ 2 & 1 & 1 & 8 \end{array}\right] \longrightarrow$$

I would like to have "1" in the first column

I want to have "1" in the upper left corner

"I want to have zero" here.

$$\xrightarrow[r_3 = r_3 - 2r_1]{r_2 = r_2 - 3r_1} \left[\begin{array}{ccc|c} 1 & -5 & 4 & 9 \\ 0 & 13 & -13 & -26 \\ 0 & 11 & -7 & -10 \end{array}\right] \xrightarrow{r_2 / : 13} \left[\begin{array}{ccc|c} 1 & -5 & 4 & 9 \\ 0 & 1 & -1 & -2 \\ 0 & 11 & -7 & -10 \end{array}\right] \xrightarrow[r_3 = r_3 - 11r_2]{r_1 = r_1 + 5r_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 4 & 12 \end{array}\right] \longrightarrow$$

I want to have "1" here

I would like to have "zero" here.

I want to have "1" here.

$$\xrightarrow{r_3 / 4} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[r_2 = r_2 + r_3]{r_1 = r_1 + r_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} \rightarrow x \\ \rightarrow y \\ \rightarrow z \end{array}$$

I would like to have "zero" here.

I_3

Now, let's check the result, comparing obtained values to my system of equations:

$$\begin{cases} 2 \cdot 2 + 1 + 3 = 8 \\ 3 \cdot 2 - 2 \cdot (1) - 3 = 1 \\ 4 \cdot 2 - 7 \cdot 1 + 3 \cdot 3 = 10 \end{cases}$$

Answer:

$x = 2$

$y = 1$

$z = 3$

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