Infinite series

Exercise 1. Try out the command: **sum 1/(n(n+1))** and have a look at the information displayed in return. In the *Infinite sum* area, we have the value of the series (in this case: *1* – so the series is convergent). Lower, in the *Convergence tests* area, you can read about the convergence tests that were applied to our series:

The ratio test is inconclusive
The root test is inconclusive
By the integral test, the series converges

Lower, in the *Partial sum formula* area, you can find the formula for the partial sum of the first *m* terms of the series – unfortunately, this information is not always displayed in case of series with more complicated formulas.

Exercise 2. Try out the following commands

sum 1/(n(n+1)), n=2..Infinity
sum Log[1-1/n^2], n=2..Infinity – unfortunately, this series turned out to be too difficult for Wolfram Alpha to provide more information about it
sum (2n-1)*(-1)^(n+1) – this series is divergent
sum Sin[(2n-1)*Pi/4] – and this one as well

Check the convergence of the following series:

$(a)\ \sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$ $(b)\ \sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$ $(c)\ \sum_{n=2}^{\infty} (\sqrt[n]{n} - \sqrt[n+1]{n+1})$ $(d)\ \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 3^n}{6^n}$ $(e)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

Exercise 3. The necessary condition for convergence of a series requires finding the limit of the general term of a series. Find the limit of the general term from the last series in the previous exercise:
limit Sin[(2n-1)*Pi/4] as n->Infinity

As a result, we got information that this limit doesn't exist – its value is not constant and belongs to the interval [-1,1].

Apply the necessary condition for convergence to the following series:

$(a)\ \sum_{n=1}^{\infty} \frac{n+2}{n+100}$ $(b)\ \sum_{n=1}^{\infty} \frac{1}{n}$ $(c)\ \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{100n^2+1}$ $(d)\ \sum_{n=1}^{\infty} \frac{n}{\ln n}$ $(e)\ \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

Exercise 4. Unfortunately, there is no command that would apply one specific test to the series. Let's have a look at this series: **sum (2^n)/(n!)**

To apply the ratio test, you should compute this limit: **limit ((2^(n+1))/((n+1)!)) * (n!/2^n) as n->Infinity**
To apply the root test, you should compute this limit: **limit ((2^n)/(n!))^(1/n) as n->Infinity**
To apply the integral test, you should compute this limit: **integrate (2^x)/(x!), x=1..Infinity**

Apply the ratio test to the following series:

$(a)\ \sum_{n=2}^{\infty} \frac{\ln n}{\pi^n}$ $(b)\ \sum_{n=1}^{\infty} \frac{2^n}{n^2}$ $(c)\ \sum_{n=1}^{\infty} \frac{(n!)^3}{(2n)!}$ $(d)\ \sum_{n=1}^{\infty} \frac{n^n}{n!}$

Apply the root test to the following series:

$(a)\ \sum_{n=1}^{\infty} (\frac{n-1}{2n+1})^n$ $(b)\ \sum_{n=1}^{\infty} \frac{3^n+4^n}{2^n+5^n}$ $(c)\ \sum_{n=1}^{\infty} \frac{n^{100}}{\pi^n}$ $(d)\ \sum_{n=1}^{\infty} n \cdot (\frac{3}{5})^n$ $(e)\ \sum_{n=1}^{\infty} (\frac{n+2}{n+3})^{n^2}$

Apply the integral test to the following series:

$(a)\ \sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $(b)\ \sum_{n=1}^{\infty} \frac{1}{4n^2+9}$ $(c)\ \sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$ $(d)\ \sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$

Exercise 5. Unfortunately, there is also no command that would check the type of convergence for the alternating series (absolute convergence, conditional convergence). You should examine each series separately:

sum n*(-1)^n/4^n – this series is convergent
sum Abs[n*(-1)^n/4^n] – and this one as well (*Abs* means the absolute value)

So, the series **sum n*(-1)^n/4^n** is absolutely convergent.

Establish the type of convergence:

$$(a)\sum_{n=1}^{\infty}\frac{(-2)^n}{n^2}\quad (b)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{\frac{3}{2}}}\quad (c)\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}\quad (d)\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\quad (e)\sum_{n=1}^{\infty}\frac{(-1)^n(n^2+2)}{n^3+3}$$

Exercise 6. Wolfram Alpha allows guessing the general term of a sequence based on the first few terms. Try out the following commands:

1+2+3+4+...
3+12+27+...
1/2 + 1/4 + 1/8 + 1/16 + ...