1. Equations and inequalities

Exercise 1a. Try out the following commands – look for the solution in *solution over the reals* :

Solve[Abs[4-x]=2,x] – x behind the last comma means it's the main variable in this equation
Solve[Abs[7-x]-Abs[8-x]+Abs[x],x]
Solve[Abs[2x+1]>3,x]
Solve[Abs[(x-1)/(x+1)]<=1,x] – look at how to write the "*less than or equal to*" sign

Exercise 1b. Solve:

a) $|4-x| = |2x+8|$, b) $|2x+6| + |8-2x| = 14$, c) $|2-x| - |3-x| = 4+x$,

Exercise 1c. Solve:

d) $\sqrt{x^2+2x+1} - 2x = 4$, e) $||2x-1|-1| \leq 1$.

Exercise 2a. Try out the following commands:

Solve[(2-x)(x+1)^2(3-x)^2<0,x]
Solve[x^3-14x^2+65x-100=0,x]
Solve[x^3-14x^2+65x-100=0,x>4.5,x] - an additional condition!
Solve[x^2+1<0,x] – answer: *no solutions exist*

Exercise 2b. Solve the system of equations:

$$\begin{cases} x^3+3x^2-4x > 12 \\ x^3-2x^2-x+2 < 0 \end{cases}$$

Exercise 2c. Solve:

a) $\sqrt{11x+6} = 8-x$, b) $\sqrt[3]{x-2} < -3$, c) $\sqrt{11-x} > x-9$, d) $3-\sqrt{x-1} = \sqrt{3x-2}$,
e) $\sqrt{x+3} + \sqrt{3x-2} \leq 7$, f) $\frac{\sqrt{x+20}}{x} < 1$, g) $\sqrt{x+5-4\sqrt{x+1}} + \sqrt{x+2-2\sqrt{x+1}} = 1$,

Exercise 3. Try out the following commands

Solve[2^x+2^(x+1)+2^(x+2)=6^x+6^(x+1),x] - look for the solution in *Real solution*
Solve[2^x=0.125,x]
Solve[(1/3)^(Abs[x-2])<= 1/9,x] – look for the solution in *Solution over the reals*

Exercise 3a. Solve:

a) $(\frac{7}{11})^{7x-11} \geq (\frac{11}{7})^{11x-7}$, b) $2^{2x} \leq 3 \cdot 2^{x+\sqrt{x}} + 4 \cdot 2^{2\sqrt{x}}$, c) $(\frac{8}{9})^{8x^2-9} \geq (\frac{9}{8})^{9x^2-8}$,

Exercise 4. Try out the following commands:

Solve[Log[3,4*3^(x-1)-1]=2x-1,x]
Solve[x^(Log[10,x])=10,x]
Solve[Abs[Log[10,x-1]+1]>=2,x]
Solve[Log[2,Log[0.5,Log[2,x]]]=0,x]

Exercise 4a. Solve:

a) $\log(x-3) - \log(27-x) \leq -\log 5 - 1$, b) $\log_{\frac{1}{3}}(|x|-1) > -2$, c) $\log_{\frac{1}{3}}(\log_5 x) \geq 0$,

Exercise 5. Test the following input:

Solve[1-2(Sin[x])^2 = (1-(Tan[x])^2)/(1+(Tan[x])^2),x]

- *Result: (all values of x are solutions)* – it's a tautology!

Exercise 5a. Check if the following equations are tautologies:

$$\text{b) } \frac{\mathrm{tg}x}{\mathrm{tg}x+\mathrm{ctg}x} = \sin^2 x, \quad \text{c) } \cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x,$$

$$\text{d) } \frac{\mathrm{tg}x+\mathrm{tg}y}{\mathrm{ctg}x+\mathrm{ctg}y} = \mathrm{tg}x \cdot \mathrm{tg}y, \quad \text{e) } \frac{1}{\sin^2 x} - 1 = \mathrm{tg}^{-2}x.$$

Exercise 6a. Solve:

$$\text{a) } \sin(\tfrac{2x-\pi}{3}) > \tfrac{1}{2}, \quad \text{b) } \cos(\tfrac{x}{2}+\pi) = \tfrac{\sqrt{2}}{2}, \quad \text{c) } \mathrm{tg}\tfrac{-x}{2} = 1, \quad \text{d) } \mathrm{ctg}\tfrac{2x+\frac{\pi}{3}}{3} > 1,$$

Exercise 6b. Solve:

$$\text{a) } \sin 3x - \sin x = 0, \quad \text{b) } 2\sin x = 3\mathrm{ctg}x, \quad \text{c) } \sin 5x - \sin 3x = 2\cos 4x,$$

$$\text{d) } \mathrm{tg}x = \sin x, \quad \text{e) } \mathrm{tg}^3 x + 1 = \mathrm{tg}^2 x + \mathrm{tg}, \quad \text{f) } \sin^4 x + \cos^4 x = \tfrac{5}{8},$$

Exercise 7. Try out the following commands:

Solve[3Acos[Abs[x^2-6x+9]]<=Pi,x] – look for the result in *Solution over the reals*

Solve[4(Asin[x])^2-Pi^2>0,x] - result: *(no solutions exist)*

Exercise 7a. Solve:

$$\text{a) } \arcsin(3-x) = \tfrac{\pi}{2}, \quad \text{b) } \arccos(\tfrac{2-x}{3}) = \pi, \quad \text{c) } \mathrm{arctg}x^3 = \tfrac{\pi}{4},$$

$$\text{d) } |3\mathrm{arcctg}x| = \pi, \quad \text{e) } 2(\arcsin x)^2 - \pi\arcsin x + \tfrac{\pi^2}{8} = 0, \quad \text{f) } \arcsin(x-1) < \tfrac{\pi}{4},$$

2. Inverse functions

Exercise 8. Try out the following commands. Look for the solutions in *Solution over the reals.*

Solve[y=2Log[4,5x-1],x]

Solve[y=Asin[2x-3],x]

Solve[y=4x+1,x]

Solve[y=Sqrt[x],x]

Solve[y=Tan[3x-Pi/4]+1,x]